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BOSTON:
PUBLISHED BY ROBERT S. DAVIS & CO.
NEW YORK: W. I. POOLEY, AND WILLIAM WOOD & CO.
PHILADELPHIA: J. B. LIPPINCOTT AND COMPANY.
ST. LOUIS: KEITH AND WOODS.
NEW ORLEANS: DUNCAN & CO.
1866.

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PREFACE.

In mental culture, no less than in manual labor, it is desirable to have instruments especially adapted to the work to be performed. Of late, owing to the circumstances of the country, it has become very evident that there are numerous learners of limited opportunities who require an easy comprehensive course of arithmetic. Such, having no time to master the several books of a consecutive series, however excellent, demand in a single book the more important principles of the science, combined with the more useful applications to ordinary business computations.

In most Graded Schools, also, there may be found a class of pupils who might advantageously use a brief elementary course, uniting mental and written exercises, before passing to any more elaborate treatise. Experience has demonstrated that beginners are best initiated into any branch by means of repeated simple illustrations and numerous easy examples.

These conditions have been carefully kept in view, during the preparation of this work. Schools have been visited in different sections, in order fully to ascertain their wants, teachers freely consulted to obtain practical suggestions, and no labor or expense spared to embody such improved methods of putting things, sanctioned by the best educators, as may not heretofore have found a place in any text-book.

The work is analytic and inductive. Mental and written exercises have been combined, with the same forms of analysis applied to both. By this plan, not only a volume of separate mental arithmetic is rendered superfluous, but valuable time is saved, and the pupil's progress promoted by an order at once natural and scientific. The number of topics treated is necessarily limited, yet will be found sufficient in range to prepare for common business.

It is proper to state that this volume has been edited by H. B. Maglathlin, A. M., who has been for many years associate editor of Greenleaf's Mathematical Series. It has been prepared as an accompaniment to that course, not to supersede any part of the same, but to supply *a new educational want*, which, heretofore, does not appear to have been exactly provided for, by any author.

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ELEMENTARY ARITHMETIC.

NUMBERS.

1. A **Unit** is a single thing of any kind, or ONE.

2. One unit and one unit are two units, or TWO.

3. Two units and one unit are three units, or THREE.

4. Three units and one unit are four units, or FOUR.

5. Four units and one unit are five units, or FIVE,

6. Five units and one unit are six units, or SIX.

7. Six units and one unit are seven units, or SEVEN.

8. Seven units and one unit are eight units, or EIGHT.

9. Eight units and one unit are nine units, or NINE.

QUESTIONS.—What is a Unit? What are one and one called? Two and one? Three and one? Four and one? Five and one? Six and one? Seven and one? Eight and one?

10. Quantity is anything that can be measured, or which has value or size. Thus,

Distance is quantity, since it can be measured, so as to be named miles, rods, etc.

11. Numbers express how many units there are in any given quantity. NUMBERS are, therefore, one or more units.

One, two, three, four, five, six, etc., are numbers.

12. Like Numbers are such as express the same kind of quantity.

Thus, five *dollars* and seven *dollars* are like numbers.

13. An **Operation** is the work done with numbers.

14. A **Rule** is the direction for performing an operation.

15. Arithmetic treats of numbers, and is the art of reckoning by them.

FIGURES.

16. Figures are certain marks, used to express numbers more briefly than by words.

Ten different figures are used in expressing numbers.

Names, or value denoted.	Figures as printed.	Figures as written.	Names, or value denoted.	Figures as printed.	Figures as written.
Cipher,	0	*0*	Five,	5	*5*
One,	1	*1*	Six,	6	*6*
Two,	2	*2*	Seven,	7	*7*
Three,	3	*3*	Eight,	8	*8*
Four,	4	*4*	Nine,	9	*9*

What is Quantity? Why is distance quantity? What do Numbers press? Mention some numbers. What are Like Numbers? What an *Operation?* A Rule? Arithmetic? What are Figures?

17. The figures 1, 2, 3, 4, 5, 6, 7, 8, 9, are called *significant* figures, because each *signifies*, or stands for, as many units as its name denotes.

18. The figure 0, or cipher, is sometimes called *zero*, or *naught*, because when used alone, it stands for *nothing*. Thus, 0 dollars means no dollars.

Exercises.

Write the following figures and name each of them:

(1)	(2)	(3)	(4)	(5)	(6)
1	8	0	7	2	9
2	9	1	2	1	7
3	6	4	8	3	8
4	3	6	8	4	6
6	2	9	5	9	0
7	4	3	4	7	5
0	5	2	1	6	4

Write in figures arranged in columns:

7. One, three, four, two, seven, five, six, eight.
8. Four, one, five, nine, eight, two, six.
9. Five, nine, two, eight, six, four, three, seven.
10. Six, cipher, three, seven, two, five, one, four.

Which figures are called significant figures? Why? Which is called zero or naught? Why?

UNITS, TENS.

19. Nine units and one unit are ten units, or TEN.

20. One ten and one unit are ELEVEN; one ten and two units are TWELVE; one ten and three units are THIRTEEN; one ten and four units are FOURTEEN; one ten and five units are FIFTEEN; one ten and six units are SIXTEEN; one ten and seven units are SEVENTEEN; one ten and eight units are EIGHTEEN; one ten and nine units are NINETEEN; "*teen*" meaning "*and ten*."

Thus, nineteen means *nine and ten*; sixteen, *six and ten*.

21. Two tens are TWENTY; three tens are THIRTY; four tens are FORTY; five tens are FIFTY; six tens are SIXTY; seven tens are SEVENTY; eight tens are EIGHTY; nine tens are NINETY; "*ty*" meaning "*tens*."

Thus, twenty means *two tens*; fifty, *five tens*; ninety, *nine tens*.

22. Exact tens, as ten, twenty, thirty, etc., are expressed by combining 1, 2, 3, etc., denoting the number of tens, with 0; and the numbers between exact tens, by combining 1, 2, 3, etc., with each of the significant figures. Thus,

Numbers requiring to be expressed by two figures are written as follows:

What is an exercise? *Ans.* A lesson for practice. What are nine and one called? What are ten and one called? Ten and two? Ten and three? Ten and four? Ten and five? Ten and six? Ten and seven? Ten and eight? Ten and nine? What is the meaning of "teen"? What are two tens called? Three tens? Four tens? Five tens? Six tens? Seven tens? Eight tens? Nine tens? What the meaning of "ty"? How are exact tens expressed? How are *numbers between* exact tens expressed?

Ten,	10
Eleven,	11
Twelve,	12
Thirteen,	13
Fourteen,	14
Fifteen,	15
Sixteen,	16
Seventeen,	17
Eighteen,	18
Nineteen,	19
Twenty,	20
Twenty-one,	21
Twenty-two,	22
Twenty-three	23
Twenty-four,	24
Twenty-five,	25
Twenty-six,	26
Twenty-seven,	27
Twenty-eight,	28
Twenty-nine,	29
Thirty,	30
Thirty-one,	31
Thirty-two,	32
Thirty-three,	33
Thirty-four,	34
Thirty-five,	35
Thirty-six,	36
Thirty-seven,	37
Thirty-eight,	38
Thirty-nine,	39
Forty,	40
Forty-one,	41
Forty-two,	42
Forty-three,	43
Forty-four,	44
Forty-five,	45
Forty-six,	46
Forty-seven,	47
Forty-eight,	48
Forty-nine,	49
Fifty,	50
Fifty-one,	51
Fifty-two,	52
Fifty-three,	53
Fifty-four,	54
Fifty-five,	55
Fifty-six,	56
Fifty-seven,	57
Fifty-eight,	58
Fifty-nine,	59
Sixty,	60
Sixty-one,	61
Sixty-two,	62
Sixty-three,	63
Sixty-four,	64
Sixty-five,	65
Sixty-six,	66
Sixty-seven,	67
Sixty-eight,	68
Sixty-nine,	69
Seventy,	70
Eighty,	80
Ninety,	90

With what figures is thirty-two written? Forty-seven? Fifty? Sixty-three?

Exercises.

Write the following figures, and read them :

(1)	(2)	(3)	(4)	(5)
10	15	11	19	91
12	23	26	55	83
14	24	28	36	75
15	25	27	57	96
17	29	22	38	98
18	30	31	50	44
20	41	22	61	89
13	39	33	65	78
21	40	34	70	99

Write in figures arranged in columns :

6. Ten, sixteen, fifty-two, eighty-eight, nineteen, seventy-three, ninety-seven, thirty-seven.

7. Thirteen, seventeen, forty-five, sixty-two, twenty-seven, forty-two, seventy-one, eighty.

8. Twenty-six, sixty-two, eighteen, eighty-one, seventy-nine, ninety-seven, fifty, five.

9. Nine, ninety, ninety-nine, thirty-five, fifty-three, eighty-two, seventy-two, twenty-two.

10. Thirty-three, forty, fifty-five, sixty-two, eighty-eight, nineteen, ninety-one.

What is the largest number that can be expressed by one figure? *By two figures? With* what figures do you write eighty? Fifty-five?

UNITS, TENS, HUNDREDS.

23. When a number is expressed by two or more figures, written together side by side, the *value* denoted by each significant figure is according to the *order* in which it is written.

Thus in 25, the 5, which is in the *first order* from the right, expresses 5 *ones*, or five; and the 2, which is in the *second order*, expresses 2 *tens*, or twenty.

24. Units expressed by a figure standing alone, or in the first order from the right, are called *primary units*, because from them are formed units of the higher orders.

Thus, by taking together ten units of the first order, and considering the group as a single thing, is formed a unit of the second order, called one ten.

25. Ten tens are called ONE HUNDRED, which forms a unit of the *third order*, and is written 100.

Exact hundreds are, therefore, expressed by writing the figure denoting their number in the third order, with two ciphers at the right. Thus, we write,

Two hundred,	200	Six hundred,	600
Three hundred,	300	Seven hundred,	700
Four hundred,	400	Eight hundred,	800
Five hundred,	500	Nine hundred,	900

According to what is the value expressed by a figure written side by side with others? What is the value of the 5 in 25? Of the 2? What are the units expressed by a figure standing alone called? Why so called? What are ten tens called? One hundred is a unit of what order? How are exact hundreds expressed? In writing a number, where do we place the figure denoting hundreds?

26. In writing a number expressed by three figures, we place the figure denoting the *hundreds* in the *third*, the figure denoting the *tens* in the *second*, and the figure denoting the *units* in the *first order*. Thus,

Two hundred and sixty-three, or 2 hundreds, 6 tens, and 3 units, is written 263.

Five hundred and seven, or 5 hundreds, 0 tens, and 7 units, is written 507.

Exercises.

Write in figures arranged in columns:

1. The numbers between one hundred and one hundred and ten.

2. The numbers between one hundred and nine and one hundred nineteen.

3. The numbers between one hundred and ninety and two hundred.

4. Three hundred, three hundred and three, three hundred and thirty, three hundred and thirty-three.

5. Five hundred and nine, nine hundred and five, five hundred and ninety, nine hundred and fifty-nine.

6. Seven hundred and seventy-seven, two hundred and twenty-two, eight hundred and eighty-eight, six hundred and six.

7. Two hundred and two, five hundred and ninety-nine, four hundred and seventy-one, nine hundred and ninety-nine.

REVIEW QUESTIONS.—What is a Unit? (1.) What is Quantity? (10.) Number? (11.) Arithmetic? (15.) Figures? (16.) What is the meaning of "teen"? (20.) Of "ty"? (21.)

Write the following figures, and read them :

(8)	(9)	(10)	(11)	(12)
112	712	443	101	199
344	341	312	397	786
106	560	707	635	552
780	818	900	563	496
920	292	601	919	999

THOUSANDS.

27. Ten hundreds are called One Thousand, which forms a unit of the *fourth* order, and is written 1000.

Exact thousands are, therefore, expressed by writing the figure denoting their number in the fourth order, with three ciphers at the right.

Thus, we write,

Two thousand,	2000	Six thousand,	6000
Three thousand,	3000	Seven thousand,	7000
Four thousand,	4000	Eight thousand,	8000
Five thousand,	5000	Nine thousand,	9000

28. Ten thousands are called One Ten-Thousand, which forms a unit of the *fifth* order, and is written 10000. Also,

Two ten-thousands, or twenty thousand, is written 20000.

Three ten-thousands, or thirty thousand, is written 30000, and so on.

29. Ten ten-thousands are called One Hundred-Thou-

What are ten hundreds called? What unit does one thousand form? How are exact thousands written? What are ten thousands called? What unit do ten thousands form? What are ten ten-thousands called?

9. The greatest number that can be expressed by three figures.

10. The greatest number that can be expressed by six figures.

NOTATION.

32. Notation is the art of writing numbers by figures, or other symbols.

The method of expressing numbers by figures is called the Arabic, because it was used by the Arabs.

33. The Arabic notation is based upon the principle, that

Ten units of any lower order are always equal to one of the next higher.

That is, ten units make one ten, ten tens make one hundred, ten hundreds make one thousand, and so on. Hence,

Each removal of a figure an order towards the left makes the value expressed ten-fold.

34. The PERIODS are named from the right; thus,

Units, thousands, millions, billions, trillions, etc.

The ORDERS are named from the right; thus,

UNITS, *tens, hundreds;* THOUSANDS, *ten-thousands, hundred-thousands;* MILLIONS, *ten-millions, hundred-millions;* BILLIONS, *ten-billions, hundred-billions;* TRILLIONS, *ten-trillions, hundred-trillions,* etc. Hence the following

---

What is Notation? What is the method of expressing numbers by figures called? Why so called? Upon what principle is the Arabic notation based? How is the value expressed increased by each removal of a figure toward the left? Name the Periods from the right. *Name the Orders.*

Table.

5th Period.			4th Period.			3d Period.			2d Period.			1st Period.		
TRILLIONS.			BILLIONS.			MILLIONS.			THOUSANDS.			UNITS.		
Hundred-trillions.	Ten-trillions.	Trillions.	Hundred-billions.	Ten-billions.	Billions.	Hundred-millions.	Ten-millions.	Millions.	Hundred-thousands.	Ten-thousands.	Thousands.	Hundreds.	Tens.	Units.
5	6	2,	7	9	3,	1	5	4,	2	0	5,	6	3	8.

where the figures express *five hundred sixty-two* TRILLION, *seven hundred ninety-three* BILLION, *one hundred fifty-four* MILLION, *two hundred five* THOUSAND, *six hundred and thirty-eight.*

The periods above Trillions, in their order, are Quadrillions, Quintillions, Sextillions, Septillions, etc.

35. To write numbers in figures.

1. Let it be required to write in figures, seven million six hundred ninety-three thousand two hundred and four.

Writing the 7 millions as the only order of the third period, the 693 thousands' as the orders of the second period, and the 204 as the orders of the first period, we have 7,693,204. Hence, the following

---

Name the periods in the Table, beginning at the right. Name the orders. Name the periods above trillions.

9. The greatest number that can be expressed by three figures.

10. The greatest number that can be expressed by six figures.

NOTATION.

32. **Notation** is the art of writing numbers by figures, or other symbols.

The method of expressing numbers by figures is called the Arabic, because it was used by the Arabs.

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Ten units of any lower order are always equal to one of the next higher.

That is, ten units make one ten, ten tens make one hundred, ten hundreds make one thousand, and so on. Hence,

Each removal of a figure an order towards the left makes the value expressed ten-fold.

34. The Periods are named from the right; thus,

Units, thousands, millions, billions, trillions, etc.

The Orders are named from the right; thus,

Units, *tens, hundreds;* thousands, *ten-thousands, hundred-thousands;* millions, *ten-millions, hundred-millions;* billions, *ten-billions, hundred-billions;* trillions, *ten-trillions, hundred-trillions,* etc. Hence the following

---

What is Notation? What is the method of expressing numbers by figures called? Why so called? Upon what principle is the Arabic notation based? How is the value expressed increased by each removal of a figure toward the left? Name the Periods from the right. *me the Orders.*

Table.

5th Period.			4th Period.			3d Period.			2d Period.			1st Period.		
TRILLIONS.			BILLIONS.			MILLIONS.			THOUSANDS.			UNITS.		
Hundred-trillions.	Ten-trillions.	Trillions.	Hundred-billions.	Ten-billions.	Billions.	Hundred-millions.	Ten-millions.	Millions.	Hundred-thousands.	Ten-thousands.	Thousands.	Hundreds.	Tens.	Units.
5	6	2,	7	9	3,	1	5	4,	2	0	5,	6	3	8.

where the figures express *five hundred sixty-two* TRILLION, *seven hundred ninety-three* BILLION, *one hundred fifty-four* MILLION, *two hundred five* THOUSAND, *six hundred and thirty-eight.*

The periods above Trillions, in their order, are Quadrillions, Quintillions, Sextillions, Septillions, etc.

35. To write numbers in figures.

1. Let it be required to write in figures, seven million six hundred ninety-three thousand two hundred and four.

Writing the 7 millions as the only order of the third period, the 693 thousands as the orders of the second period, and the 204 as the orders of the first period, we have 7,693,204. Hence, the following

---

Name the periods in the Table, beginning at the right. Name the orders. Name the periods above trillions.

Rule.

Beginning with the highest period to be expressed, write the figures belonging to each period, in their orders, observing to mark the omission of any order of units with a cipher.

Examples.

Write in figures the following numbers:

2. Seven hundred and seventy.
3. One thousand eight hundred and eighty-five.
4. Three thousand five hundred and fifty-three.
5. Eleven thousand and one.
6. Eleven hundred and eleven.
7. Seventy-three thousand five hundred and ninety-two.
8. Eighty-four thousand nine hundred and nine.
9. Two hundred thirty thousand five hundred and six.
10. Forty-one thousand and nineteen.
11. Nine thousand nine hundred and seven.
12. Eighty-nine thousand and ninety-seven.
13. Twenty-one thousand one hundred and twenty-one.
14. Three hundred thousand and six.
15. Fifteen thousand one hundred and fifteen.
16. Seventy-nine thousand nine hundred and seven.
17. Sixty-seven thousand three hundred and six.
18. Six hundred thirty-five thousand four hundred and thirty-eight.

Repeat the Rule. What is an Example? *Ans.* A question, or *mething to be done,* under a rule.

19. Forty-two thousand four hundred and forty-four.

20. Ninety-eight thousand six hundred and nine.

21. Nineteen thousand three hundred and fifty-one.

22. One hundred thousand and forty-seven.

23. One million ten thousand and ten.

24. Sixty-one million sixteen thousand six hundred and five.

25. Eight hundred twelve thousand three hundred and forty-seven.

26. Twelve million twenty thousand three hundred and one.

27. Seven million nine hundred twenty-three thousand four hundred and six.

28. Three billion one hundred eleven million two hundred twenty thousand and two.

29. Five hundred eighty-one million thirty-six thousand and twenty-nine.

30. One trillion one million one thousand and ninety-one.

31. Twenty-nine million fifty thousand one hundred and fifty.

32. One hundred million one hundred thousand one hundred and one.

33. Six hundred thirty-one million one hundred twenty-four thousand and sixty-six.

34. Five trillion five thousand and five.

35. Two hundred ninety trillion six hundred thirty billion four hundred two million four hundred seventy-nine thousand eight hundred and fifteen.

Review Questions.—What is an Operation? (13.) What is a Rule? (14.) What is the cipher sometimes called? (18.) What do ciphers written with other figures denote, as in 108, 240, etc.? Ans. Vacant orders, or such as have an omission of units.

NUMERATION.

36. Numeration is the art of reading numbers expressed by figures.

37. To read numbers expressed by figures.

1. Let it be required to read 7693254.

Pointing off the given figures into periods, from the right, we have 7,693,254.

The third period is 7 *millions*, the second period is 693 *thousands*, the first period is 254 *units;* therefore, the whole reads, seven million six hundred ninety-three thousand two hundred and fifty-four. Hence, the following

Rule.

Beginning at the right, point off the given expression into as many periods as possible, of three figures each.

Then, begin at the left and read each period, giving after each, excepting the units, the name of the period.

Examples.

Point off and read the following:

2. 932
3. 1865
4. 7060
5. 11001
6. 1199
7. 73592
8. 170643
9. 230506
10. 31031
11. 9907
12. 73421
13. 1306781

What is Numeration? At which hand do you begin to point off numbers? At which to read? Repeat the Rule. Why is the name of the units' period not given in reading figures? *Ans. Because the name is understood.*

13.	214574	18.	2923406	23.	13412
14.	300006	19.	22443	24.	712345
15.	21034	20.	3992	25.	814347
16.	71290	21.	17700	26.	91628
17.	635438	22.	100047	27.	12020301

28.	631245687	31.	78900672164
29.	123000000000	32.	130132456987
30.	444555833777	33.	29630402479815

ADDITION.

38. If James has 4 books and John 2, how many books have both of them?

Solution. *If James has* 4 *books and John* 2, *both of them have* 4 *books and* 2 *books, which are* 6 *books. Therefore, both of them have* 6 *books.*

The preceding operation, which consists in finding a single number containing the same number of units as the two given numbers of the same kind, is called Addition. Hence,

39. Addition is the process of finding a number equal to two or more given numbers of the same kind.

The Sum or Amount is the number which is equal to all the numbers added.

Thus, the sum of 2 cents and 3 cents, is 5 cents.

In what does the operation consist? What is it called? What is Addition? The Sum or Amount? What is a Solution? *Ans.* The answering of a question, or giving in order the steps required to obtain its answer.

Addition Table.

1 and	2 and	3 and	4 and	5 and
1 are 2	1 are 3	1 are 4	1 are 5	1 are 6
2 " 3	2 " 4	2 " 5	2 " 6	2 " 7
3 " 4	3 " 5	3 " 6	3 " 7	3 " 8
4 " 5	4 " 6	4 " 7	4 " 8	4 " 9
5 " 6	5 " 7	5 " 8	5 " 9	5 " 10
6 " 7	6 " 8	6 " 9	6 " 10	6 " 11
7 " 8	7 " 9	7 " 10	7 " 11	7 " 12
8 " 9	8 " 10	8 " 11	8 " 12	8 " 13
9 " 10	9 " 11	9 " 12	9 " 13	9 " 14
10 " 11	10 " 12	10 " 13	10 " 14	10 " 15
6 and	**7 and**	**8 and**	**9 and**	**10 and**
1 are 7	1 are 8	1 are 9	1 are 10	1 are 11
2 " 8	2 " 9	2 " 10	2 " 11	2 " 12
3 " 9	3 " 10	3 " 11	3 " 12	3 " 13
4 " 10	4 " 11	4 " 12	4 " 13	4 " 14
5 " 11	5 " 12	5 " 13	5 " 14	5 " 15
6 " 12	6 " 13	6 " 14	6 " 15	6 " 16
7 " 13	7 " 14	7 " 15	7 " 16	7 " 17
8 " 14	8 " 15	8 " 16	8 " 17	8 " 18
9 " 15	9 " 16	9 " 17	9 " 18	9 " 19
10 " 16	10 " 17	10 " 18	10 " 19	10 " 20

40. A Sign is a mark used to denote an operation.

The Sign of Addition is an erect cross, +, called *plus*. Thus, 3 + 2, read three plus two, denotes that 3 and 2 are to be added.

Repeat the column 1 and 1. 2 and 1. 3 and 1. 4 and 1. 5 and 1. 6 and 1. 7 and 1. 8 and 1. 9 and 1. 10 and 1. What is a sign? *What is the sign of* Addition?

The Sign of Equality is two short parallel horizontal lines, read equals, or equal to. Thus, $3+2=5$, is read, three plus two are equal to five.

Exercises on the Table.

1. 2 and 4 are how many?
2. 5 and 1 are how many?
3. 4 and 3 are how many?
4. 3 and 7 are how many?
5. 8 and 2 are how many?
6. 6 and 5 are how many?
7. 9 and 6 are how many?
8. 3 and 8 are how many?
9. 3 and 5 are how many?
10. 7 and 4 are how many?
11. 6 and 6 are how many?
12. 8 and 5 are how many?
13. 4 and 9 are how many?
14. 9 and 2 are how many?
15. 5 and 7 are how many?
16. 8 and 8 are how many?

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17. $2+6$ are how many?
18. $7+7$ are how many?
19. $3+3$ are how many?
20. $5+9$ are how many?
21. $6+7$ are how many?
22. $8+1$ are how many?
23. $4+4$ are how many?
24. $1+9$ are how many?
25. $5+2$ are how many?
26. $7+9$ are how many?
27. $6+3$ are how many?
28. $4+6$ are how many?
29. $8+6$ are how many?
30. $9+9$ are how many?

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31. $3+9=$ how many?
32. $4+5=$ how many?
33. $10+3=$ how many?
34. $5+5=$ how many?
35. $8+10=$ how many?
36. $7+10=$ how many?
37. $7+1=$ how many?
38. $8+4=$ how many?
39. $1+10=$ how many?
40. $10+6=$ how many?
41. $9+10=$ how many?
42. $10+10=$ how many?

What is the sign of Equality?

Review Questions. What is Notation? (32). Numeration? (36). What is the Rule for writing numbers? (35). For reading numbers? (37).

43. How many are 13 and 4? 23 and 4? 63 and 4? 93 and 4? 15 and 6? 35 and 6? 6 and 55?

44. How many are 11 and 7? 21 and 7? 51 and 7? 71 and 7? 12 and 3? 32 and 3? 83 and 3?

45. How many are 16 and 2? 46 and 2? 46 and 5? 66 and 5? 96 and 5? 17 and 3? 37 and 3?

46. How many are 18 and 7? 28 and 7? 68 and 7? 98 and 7? 14 and 7? 27 and 7?

47. How many are 19 and 8? 29 and 8? 69 and 8? 89 and 9? 10 and 9? 20 and 9? 80 and 9? 90 and 9?

Mental Exercises.

1. John has 4 apples, Arthur 3, and Henry 2; how many have they in all?

SOLUTION. *They have the sum of* 4 *apples,* 3 *apples, and* 2 *apples;* 4 *and* 3 *are* 7, *and* 2 *are* 9. *Therefore, they have in all* 9 *apples.*

2. Paid for paper 8 cents, and for pencils 6; how much was paid for the whole?

3. James caught 7 fishes, Ernest 5, and Albert 6; how many were caught by them all?

4. Gave 9 dollars for coal, and 11 dollars for wood; how many dollars were given for both?

5. If a writing-book cost 10 cents, a pen 5 cents, and a slate 9 cents, how much do they all cost?

6. A clothier sold a hat for 5 dollars, a vest for 9, and a coat for as much as he sold both the hat and vest for; for how much was the coat sold?

7. Jane has 5 books at school, 7 at home, and has loaned 6; how many has she in all?

REVIEW QUESTIONS. What is the method of expressing numbers *by figures called? (32).*

and note the absence of any order of units, by a cipher Thus,

Five thousand and twenty, or 5 thousands no hundreds 2 tens no units, is written 5,020.

Fifty thousand two hundred and two, or 5 ten-thousands 2 hundreds 2 units, is written 50,202.

Five hundred and fifty-five thousand two hundred and twenty-two, or 5 hundred-thousands 5 ten-thousands 5 thousands 2 hundreds 2 tens 2 units, is written 555,222.

Exercises.

Write in figures and read.

1. Six units of the fifth order, with seven units of the third order.

Ans. 60,700; read, sixty thousand and seven hundred.

2. 3 hundred-thousands 4 ten-thousands and 4 thousands.

Ans. 344,000 ; read, three hundred forty-four thousand.

3. Two units of the fourth order, with three units of the second order.

4. Eight units of the fifth order, with three units of each of the lower orders.

5. 9 units of the sixth order, with no units of the fifth order, and six units of each of the lower orders.

6. 316 in the second period, and 0 in each order of the first period.

7. 21 thousands 2 tens and 1 unit.

8. 2 hundred-thousands 5 ten-thousands with five units of the third order.

REVIEW QUESTIONS. — What are ten tens called ? (25.) Ten hundreds ? (27.) Ten thousands ? (28) Ten ten-thousands ? (29.) Name the first six orders. (30.) The first two periods (30.)

2. Required the sum of 595, 361, and 723.

OPERATION.
595
361
723
Sum, 1679

For convenience we write, as before, the figures of the same order in the same column, and begin with units to add.

3, 1, and 5 units are 9 *units*; we write the 9. 2, 6, and 9 are 17 *tens*, or 1 hundred and 7 tens; we write the 7, and add the 1 in with hundreds. 1, 7, 3, and 5 hundreds are 16 *hundreds*, or 1 thousand and 6 hundreds, which we write. Therefore, the sum required is 1 thousand, 6 hundreds, 7 tens, and 9 units, or 1679.

In practice, it is sufficient to name only results. Thus, in the operation we may say: three, four, *nine*, — write 9; two, eight, *seventeen*, — write 7 and add 1 with next column; eight, eleven, *sixteen*, — write 16; answer, 1679.

Rule.

Write the numbers to be added so that figures of the same order shall stand in the same column.

Beginning with units, add each column separately, and write the sum underneath, if less than ten of the order added.

If, however, the sum is ten or more, write the right-hand figure, and add the number expressed by the other figure or figures with the numbers of the next column.

Write the whole sum of the last column.

Why is it convenient to have figures of the same order in the same column? *Ans.* Because, only units of the same order can be directly added.

How do you write the numbers to be added? How do you add each column? What do you do with the sum of each column? *Repeat the Rule.*

PROOF. Add the numbers a second time, but in such an order that they shall have been added the one time upwards and the other downwards. If the work is correct, the result will be the same by both methods.

Examples.

(3.)	(4.)	(5.)	(6.)	(7.)
216	452	501	691	334
103	123	120	10	160
520	211	17	9	123
839	786	638	710	617

(8.)	(9.)	(10.)	(11.)	(12.)
610	317	491	216	929
291	812	870	725	442
208	402	143	140	312

(13.)	(14.)	(15.)	(16.)	(17.)
621	723	991	31	818
333	451	61	615	98
998	689	781	123	3

How is addition proved? What is the reason for this proof? *Ans.* The same error, if any, is not likely to be repeated, when the same numbers are added in a different order.

Why do we find it convenient to begin with units to add? *Ans.* Because, if the sum of any column is greater than nine of the order added, we can readily add in its left-hand figure or figures with the next column.

(18.)	(19.)	(20.)	(21.)	(22.)
110	72	402	331	928
99	491	40	617	713
6	149	449	100	456

(23.)	(24.)	(25.)	(26.)
1781	206	125	9020
121	312	39	3451
5642	871	620	7770
2401	462	510	1223

(27.) Dollars.	(28.) Years.	(29.) Miles.	(30.) Bushels.
1234	7261	2168	2341
5610	345	1681	5123
1432	69	2002	4402

31. What is the sum of 9401, 105, and 47?
Ans. 9553.

32. What is the sum of 324 + 643 + 201 + 46?
Ans. 1214.

33. What is the sum of 5684 + 340 + 7006 + 309 + 824?

34. 170 + 640 + 126 + 123 + 11 = how many?
Ans. 1070.

35. 36 + 320 + 708 + 17 + 3 = how many?
Ans. 1084.

36. Find the sum of 31, 114, 3047, and 57183.
Ans. 60375.

When the sum of a column is ten or more, why is the right-hand figure written underneath? *Ans.* Because it expresses units *of the order* added.

37. Find the sum of 19, 119, 11919, and 9191.

38. A farmer has four fields; the first yields 1109 bushels of corn, the second 301, the third 516, and the fourth 613; how many bushels do the four fields yield? Ans. 2539 bushels.

39. George Washington was born in the year 1732, and lived 67 years. In what year did he die?

40. In January there are 31 days, February 28, March 31, April 30, May 31, and June 30; how many days are there in these six months? Ans. 181 days.

41. I have bought six lots of coal, consisting of 210, 317, 258, 610, 516, and 311 tons, respectively; how many tons are there in all? Ans. 2222 tons.

42. What is the sum of nine hundred three, four thousand fourteen, fifteen, sixty-three, and three thousand three hundred two? Ans. 8297.

43. What is the sum of fifteen thousand fifteen, one thousand six hundred one, ten thousand one hundred ten, and seven hundred seventeen? Ans. 27443.

44. Paid for a farm 6715 dollars for having it fenced 635 dollars, and for having a barn built upon it 500 dollars. For how much must it be sold to gain 200 dollars? Ans. 8050 dollars.

45. What is the extent of the British Isles, if Scotland has 30000 square miles, England 51200, Wales 7200, and Ireland 32500? Ans. 120900 square miles.

46. The United States is 3385000 square miles in extent, Brazil 3000000, Russia in Europe 2100000, France 204800, and Austria 259300; what is the extent of all these countries? Ans. 8949100 square miles.

Why is the left-hand figure or figures of the sum of a column added in with the next column at the left? *Ans.* Because, ten units of the order added are one of the order next higher.

Long Columns.

Practice upon the following examples, until the numbers of each can be added correctly and rapidly, by naming results only. (Art. 41.)

(47.)	(48.)	(49.)	(50.)	(51.)
63	44	123	316	150
57	36	503	134	529
91	28	175	111	271
36	33	497	132	114
86	55	502	436	497
45	70	212	235	320
35	34	684	147	169
413		2696		2050

(52.)	(53.)	(54.)	(55.)	(56.)
307	21	746	20	173
133	86	672	93	813
810	24	336	18	324
341	14	187	78	407
458	43	267	34	611
817	54	164	27	710
802	73	928	57	206
863	32	748	26	502
4531		4048		3746

Review Questions. What is Addition? (39.) What is the *esult in Addition called?* (39.) Repeat the Rule for Addition. (41.)

SUBTRACTION.

42. If John has 6 apples and gives his brother 4 of them, how many will he have left?

SOLUTION. *If John has* 6 *apples and gives his brother* 4 *of them, he will then have left the difference between* 6 *apples and* 4 *apples, which is* 2 *apples. Therefore, he will have left* 2 *apples*

This operation consists in finding the difference between the two given numbers, by taking the smaller from the larger, and is called *subtraction*. Hence,

43. Subtraction is the process of finding the difference between two numbers of the same kind.

The DIFFERENCE, or REMAINDER, is the number left after the subtraction is performed.

Thus, the difference between 6 apples and 4 apples is 2 apples.

44. The SUBTRAHEND is the number subtracted.

45. The MINUEND is the number subtracted from.

46. The DIFFERENCE between two numbers is such a number as added to the smaller will equal the larger; or, such a number as subtracted from the larger will equal the smaller.

Thus, the difference between 6 apples and 4 apples is 2 apples; and 2 apples added to the 4 apples is equal to 6 apples; or, 2 apples subtracted from 6 apples is equal to the 4 apples.

In what does the operation consist? What is Subtraction? The Difference? The Subtrahend? The Minuend? The Difference between two Numbers?

Subtraction Table.

1 from	2 from	3 from	4 from	5 from
1 leaves 0	2 leaves 0	3 leaves 0	4 leaves 0	5 leaves 0
2 " 1	3 " 1	4 " 1	5 " 1	6 " 1
3 " 2	4 " 2	5 " 2	6 " 2	7 " 2
4 " 3	5 " 3	6 " 3	7 " 3	8 " 3
5 " 4	6 " 4	7 " 4	8 " 4	9 " 4
6 " 5	7 " 5	8 " 5	9 " 5	10 " 5
7 " 6	8 " 6	9 " 6	10 " 6	11 " 6
8 " 7	9 " 7	10 " 7	11 " 7	12 " 7
9 " 8	10 " 8	11 " 8	12 " 8	13 " 8
10 " 9	11 " 9	12 " 9	13 " 9	14 " 9
11 " 10	12 " 10	13 " 10	14 " 10	15 " 10

6 from	7 from	8 from	9 from	10 from
6 leaves 0	7 leaves 0	8 leaves 0	9 leaves 0	10 leaves 0
7 " 1	8 " 1	9 " 1	10 " 1	11 " 1
8 " 2	9 " 2	10 " 2	11 " 2	12 " 2
9 " 3	10 " 3	11 " 3	12 " 3	13 " 3
10 " 4	11 " 4	12 " 4	13 " 4	14 " 4
11 " 5	12 " 5	13 " 5	14 " 5	15 " 5
12 " 6	13 " 6	14 " 6	15 " 6	16 " 6
13 " 7	14 " 7	15 " 7	16 " 7	17 " 7
14 " 8	15 " 8	16 " 8	17 " 8	18 " 8
15 " 9	16 " 9	17 " 9	18 " 9	19 " 9
16 " 10	17 " 10	18 " 10	19 " 10	20 " 10

47. The Sign of Subtraction is a short horizontal line, —, called *minus*. Thus, 6 — 4, read six minus four, denotes that 4 is to be subtracted from 6.

What is the Sign of Subtraction? Repeat the column 1 from 1. 2 from 2. 3 from 3. 4 from 4. 5 from 5. 6 from 6. 7 from 7. 8 from 8. 9 from 9. 10 from 10.

Exercises on the Table.

1. 2 from 6 how many?
2. 1 from 8 how many?
3. 3 from 7 how many?
4. 8 from 10 how many?
5. 6 from 11 how many?
6. 9 from 15 how many?
7. 3 from 11 how many?
8. 7 from 10 how many?
9. 3 from 8 how many?
10. 7 from 11 how many?
11. 6 from 12 how many?
12. 5 from 13 how many?
13. 9 from 13 how many?
14. 2 from 11 how many?
15. 7 from 12 how many?
16. 8 from 16 how many?

17. 6 from 8 how many?
18. 7 from 14 how many?
19. 3 from 6 how many?
20. 5 from 14 how many?
21. 6 from 13 how many?
22. 8 from 9 how many?
23. 4 from 4 how many?
24. 2 from 11 how many?
25. 5 from 17 how many?
26. 9 from 16 how many?
27. 2 from 10 how many?
28. 7 from 16 how many?
29. 9 from 18 how many?
30. 6 from 14 how many?

31. 13 — 4 = how many?
32. 9 — 5 = how many?
33. 13 — 5 = how many?
34. 16 — 6 = how many?
35. 17 — 10 = how many?
36. 18 — 9 = how many?
37. 8 — 2 = how many?
38. 14 — 8 = how many?
39. 11 — 1 = how many?
40. 16 — 7 = how many?
41. 19 — 10 = how many?
42. 20 — 10 = how many?

43. How many does 6 from 13 leave? 6 from 23? 6 from 43? 6 from 73? 6 from 93?

Review Questions.—What is the sign of Subtraction called? (47.) Sign of Addition? (40.)

44. How many does 5 from 12 leave? 5 from 32? 5 from 22? 5 from 42? 5 from 72?

45. How many does 7 from 11 leave? 7 from 21? 7 from 41? 7 from 64? 7 from 84?

46. How many does 8 from 16 leave? 8 from 36? 8 from 76? 8 from 96? 8 from 15? 8 from 55?

47. How many does 9 from 17 leave? 9 from 27? 9 from 37? 9 from 67? 9 from 77? 9 from 88?

48. How many does 3 from 10 leave? 3 from 40? 3 from 60? 3 from 70? 3 from 90?

Mental Exercises.

1. Mary had 8 books, and gave to her sister 3 of them; how many books had she left?

SOLUTION. *She had left the difference between* 8 *books and* 3 *books, and* 3 *from* 8 *leaves* 5. *Therefore, she had* 5 *books left.*

2. If a boy has 13 marbles and gives away 7 of of them, how many has he left?

3. Bought a watch for $10 and sold it for 17; how much was gained?

4. From a brood of 15 chickens the hawks have taken 6; how many remain?

5. If I go a-shopping with 18 cents, and spend 8 cents, how much have I left?

6. Sold goods for 21 dollars, which was 5 dollars more than they cost me; what did they cost me?

7. Bought a cow for 31 dollars, and paid down 9 dollars; how much was then owed?

REVIEW QUESTIONS. — What are Numbers? (11.) Figures? *(16.) How many are used?* (16.)

8. Andrew has 4 cents, and wishes to buy a knife worth 42 cents; how many more cents does he need?

9. 23 from 47 leaves how many?

SOLUTION.—*As many as the difference between* 47 *and* 23; 47 *is* 4 *tens and* 7 *units, and* 23 *is* 2 *tens and* 3 *units;* 2 *tens from* 4 *tens leaves* 2 *tens, and* 3 *units from* 7 *units leaves* 4 *units; and* 2 *tens, or* 20 *units, and* 4 *units are* 24. *Therefore,* 23 *from* 47 *leaves* 24.

10. 17 from 28 leaves how many? 19 from 39?

11. 25 from 46 leaves how many? 35 from 55?

12. There are 21 pupils in one school-room, and 33 in another; how many less are there in the one than in the other?

13. A merchant bought at one time 58 chests of tea, and at another 34; how many more did he buy at one time than at the other?

14. A farmer sold 27 bushels of wheat from a bin containing 49 bushels; how many bushels were left in the bin?

Written Exercises.

48. 1. Let it be required to subtract 325 from 958.

OPERATION.

Minuend, 958
Subtrahend, 325
Difference, 633

For convenience, we write the subtrahend under the minuend, so that figures of the same order stand in the same column, and begin at the right to subtract.

5 units from 8 units leave 3 units, which we write underneath. 2 tens from 5 tens

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How do you write the given numbers? Why is it convenient to have the figures of the subtrahend written under figures of the same order in the minuend? *Ans.* Because, only units of the same order can be subtracted?

leave 3 tens, which we write. 3 hundreds from 9 hundreds leave 6 hundreds, which we write. Therefore, the difference required is 6 hundreds, 3 tens, and 3 units, or 633.

2. Let it be required to find the difference between 652 and 423.

OPERATION.

Minuend,	652
Subtrahend,	423
Difference,	229

For convenience, we write, as before, figures of the same order in the same column, and begin with units to subtract.

We cannot take 3 units from 2 units; but we can take 1 ten from the 5 tens, leaving 4 tens; and the 1 ten taken is 10 units, which added to the 2 units make 12 units; 3 units from 12 units leave 9 units, which we write underneath. 2 tens from 4 tens leave 2 tens, which we write. 4 hundreds from 6 hundreds leave 2 hundreds, which we write. Therefore, the difference required is 2 hundreds, 2 tens, and 9 units, or 229.

The operation may be performed another way: as the 3 units cannot be taken from 2 units, we add 10 units to the 2 units; and 3 units from 12 units leave 9 units, which we write. To balance the 10 units added to the 2 units, we add 1 ten to the 2 tens, in accordance with the principle, that *if any two numbers are equally increased their difference remains the same;* and say, 3 tens from 5 tens leave 2 tens, which we write. 4 hundreds from 6 hundreds leave 2 hundreds, which we write. Therefore, the difference is 229.

In practice, a briefer explanation is sufficient. Thus, we may say: 3 from 2, impossible; but 3 from 12 leaves 9; we write 9, and add 1 to the next figure; 1 plus 2 are 3, and 3 from 5 leaves 2, which we write; 4 from 6 leaves 2, which we write. Answer, 229.

When a figure of the subtrahend is greater than the figure of the minuend above it, what do you do? Mention another way. According to what principle is the 10 added to a figure of the minuend, balanced by the addition of 1 to the next left hand figure of the subtrahend?

Rule.

Write the less number under the greater, so that units of the same order shall stand in the same column.

Beginning with units, subtract each order of the subtrahend from the order above it, and write the difference underneath.

If the number of any order of the subtrahend be greater than that of the order above it, conceive the number of the upper order to be increased by ten, and the number of the next order of the minuend to be diminished by one, or the next order of the subtrahend increased by one, and proceed as before.

Proof. Add the difference to the subtrahend, and, if the work is correct, the sum will equal the minuend ; or,

Subtract the difference from the minuend, and if the work is correct, what is left will equal the subtrahend.

Examples.

	(3.)	(4.)	(5.)
Minuend,	466	673	2006
Subtrahend,	270	564	107
Difference,	196	109	1899
Proof,	466	673	2006

	(6.)	(7.)	(8.)	(9.)
From	890	526	812	723
Subtract	763	19	102	16

How are the given numbers written for subtraction ? Where do you begin to subtract ? If a figure of the subtrahend is greater than the figure above it, what do you do ? Repeat the Rule. What is the method of proof ?

	(10.)	(11.)	(12.)	(13.)
From	423	902	876	200
Take	310	713	575	189

	(14.)	(15.)	(16.)	(17.)
From	333	900	776	307
Take	244	109	445	206

	(18.)	(19.)	(20.)	(21.)
From	2006	2110	1008	4708
Take	1341	1102	19	1080

	(22.)	(23.)	(24.)	(25.)
	Dollars.	Miles.	Pounds.	Yards.
From	7987	4162	7910	8008
Take	890	1051	6820	909
	7077	3111	1090	7099

26. From 1937 tons subtract 1128 tons.
Ans. 809 tons.
27. From 1000 barrels take 98 barrels.
Ans. 902 barrels.
28 463 cords less 209 cords = how many cords?
29. 3612 rods — 2004 rods = how many rods?
Ans. 1608 rods.
30. 21020 — 16101 = how many? Ans. 4919.
31. 30000 — 1010 = how many? Ans. 28990.
32. 111111 — 11112 = how many? Ans. 99999.
33. Subtract 635 from 34621. Ans. 33986.

REVIEW QUESTIONS. — What is the sign of Subtraction? (47.) *What is the sign of Equality?* (40.)

34. Subtract 31012 from 31201. Ans. 189.

35. Take 416 from 928. Ans. 512.

36. Take 313142 from 404030. Ans. 90888.

37. I had 1630 dollars in the bank, but have drawn out 635 dollars. How many dollars have I remaining in the bank? Ans. 995 dollars.

38. Paid 225 dollars for a horse and 165 dollars for a buggy; how much more was paid for the horse than for the buggy? Ans. 60 dollars.

39. How old was a man in 1865, who was born in 1819?

40. The Pilgrims landed at Plymouth in 1620; how long after was the Declaration of Independence, which was in 1776? Ans. 156 years.

41. John Jones bought a farm for 4000 dollars, and has paid for it within 719 dollars. How much has he paid already? Ans. 3281 dollars.

42. Maryland has 9356 square miles, and Delaware 2120 square miles; how many more square miles has the one than the other? Ans. 7236.

43. A farmer raised 6723 bushels of corn, and has sold 3846 bushels; how much remains unsold?

44. From ten thousand ten, take one thousand one hundred and one. Ans. 8909.

45. From sixteen thousand five hundred and ninety, take twelve thousand nine hundred and nine.
Ans. 3681.

46. A trader bought 30 casks of cider, containing 1890 gallons, and sold 23 casks, containing 1449 gal-

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Review Questions.—What is Addition? (39.) What is the Sum? (39.) Of what name or kind is the sum? *Ans.* The same as that of the numbers added.

lons. How many casks, and how many gallons, were left? Ans. 7 casks; 441 gallons.

47. The salary of the President in four years amounts to one hundred thousand dollars; if his expenses in that time are 63575 dollars, how much of his salary will be left? Ans. 36425 dollars.

48. Albert Smith sold in one year goods to the amount of 58692 dollars, which was 5863 dollars more than he gave for them. How much did he give for the goods? Ans. 52829 dollars.

49. The smaller of two numbers is 61342, and the larger 1000300; what is their difference?

50. If the mean distance of the sun from the earth is ninety-five million one hundred seventy-three thousand one hundred and twenty-seven miles, and that of the moon two hundred thirty-eight thousand six hundred and fifty miles, how much farther is the sun than the moon from the earth?

Ans. 94934477 miles.

Examples Combining Addition and Subtraction.

1. A man having 7839 dollars, paid away at one time 317 dollars, and at another 2163; how much did he then have left?

OPERATION.

2163	7839
317	2480
——	——
2480	5359

He had left the difference between 7839 dollars and the sum of 2163 dollars and 317 dollars; the sum of 2163 dollars and 317 dollars is 2480 dollars; and the difference between 7839 dollars and 2480 dollars is 5359 dollars. Therefore, he had left 5359 dollars.

Review Questions. — What is Subtraction? How does it differ *from Addition?* *Ans.* It is the reverse of Addition.

2. Bought flour to the amount of 256 dollars, cloth to the amount of 317 dollars, and paid down 373 dollars; how much remained to be paid?

Ans. 200 dollars.

3 Henry Smith went out with 435 dollars to settle accounts. He paid 115 dollars to one man, received 119 dollars from another and paid 316 to another. How much had he left? Ans. 123 dollars.

4. A boy had 216 apples; he sold 34 of them one day, 72 another day, and afterwards 29; how many had he left?

5. Bought a house for 3165 dollars, laid out for repairs on it 593 dollars, and sold it for 3500 dollars; how much was lost by the bargain? Ans. 258 dollars.

6. Sold cloth for 10 dollars, sugar 17 dollars, pork 43 dollars, beef 19 dollars, and fish 13 dollars; and in payment received a check for 75 dollars, and a 50-dollar note. How much change should be paid back?

Ans. 23 dollars.

7. A farmer buys a yoke of oxen for 163 dollars. He exchanges them for others, paying 27 dollars, and then sells for 200 dollars. Did he gain or lose?—and how much?

8. Dana estimates that there are workable coal measures in the United States to the extent of 130000 square miles; and in Great Britain and Ireland of 12000, in Spain of 4000, in France of 2000, and in Belgium of 518. How much do those of the United States exceed those of the other countries named?

Ans. 111482 square miles.

Of what name or kind is the answer in Subtraction? *Ans.* The same as that of the Minuend and Subtrahend? What is the Rule for Subtraction? Proof?

MULTIPLICATION.

49. If a boy can earn 5 dollars in one week, how many dollars can he earn in 4 weeks ?

SOLUTION. *If a boy can earn* 5 *dollars in one week, he can earn in* 4 *weeks* 4 *times* 5 *dollars, which are* 20 *dollars. Therefore, if he can earn* 5 *dollars in one week, he can earn in* 4 *weeks* 20 *dollars.*

Here the process consists in taking the 5 dollars as many times as the given number of weeks, or 4 times, and is called Multiplication. Hence,

50. Multiplication is the process of taking one number as many times as there are units in another number.

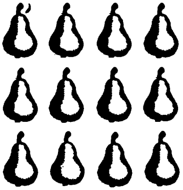
51. The PRODUCT is the result of the multiplication.

52. The MULTIPLICAND is the number to be taken.

53. The MULTIPLIER is the number denoting how many times the multiplicand is to be taken.

54. The FACTORS are the multiplier and multiplicand.

55. The PRODUCT of two or more factors is the same in whatever order they are taken.



Thus, the product of 4 by 3, or 3 by 4, is 12. This is shown by the engraving. The number of pears in the picture are the same, whether we consider them 4 in a row and 3 rows, or 3 in a row and 4 rows.

Of what does the process of the solution consist ? What is Multiplication ? The Product ? What is the Multiplicand ? What is the Multiplier ? The Factors ? Show that the product of two *factors is the same in* whatever order taken.

Multiplication Table.

Once	2 times	3 times	4 times	5 times	6 times
1 is 1	1 are 2	1 are 3	1 are 4	1 are 5	1 are 6
2 " 2	2 " 4	2 " 6	2 " 8	2 " 10	2 " 12
3 " 3	3 " 6	3 " 9	3 " 12	3 " 15	3 " 18
4 " 4	4 " 8	4 " 12	4 " 16	4 " 20	4 " 24
5 " 5	5 " 10	5 " 15	5 " 20	5 " 25	5 " 30
6 " 6	6 " 12	6 " 18	6 " 24	6 " 30	6 " 36
7 " 7	7 " 14	7 " 21	7 " 28	7 " 35	7 " 42
8 " 8	8 " 16	8 " 24	8 " 32	8 " 40	8 " 48
9 " 9	9 " 18	9 " 27	9 " 36	9 " 45	9 " 54
10 " 10	10 " 20	10 " 30	10 " 40	10 " 50	10 " 60
11 " 11	11 " 22	11 " 33	11 " 44	11 " 55	11 " 66
12 " 12	12 " 24	12 " 36	12 " 48	12 " 60	12 " 72

7 times	8 times	9 times	10 times	11 times	12 times
1 are 7	1 are 8	1 are 9	1 are 10	1 are 11	1 are 12
2 " 14	2 " 16	2 " 18	2 " 20	2 " 22	2 " 24
3 " 21	3 " 24	3 " 27	3 " 30	3 " 33	3 " 36
4 " 28	4 " 32	4 " 36	4 " 40	4 " 44	4 " 48
5 " 35	5 " 40	5 " 45	5 " 50	5 " 55	5 " 60
6 " 42	6 " 48	6 " 54	6 " 60	6 " 66	6 " 72
7 " 49	7 " 56	7 " 63	7 " 70	7 " 77	7 " 84
8 " 56	8 " 64	8 " 72	8 " 80	8 " 88	8 " 96
9 " 63	9 " 72	9 " 81	9 " 90	9 " 99	9 " 108
10 " 70	10 " 80	10 " 90	10 " 100	10 " 110	10 " 120
11 " 77	11 " 88	11 " 99	11 " 110	11 " 121	11 " 132
2 " 84	12 " 96	12 " 108	12 " 120	12 " 132	12 " 144

56. The SIGN OF MULTIPLICATION is an inclined cross, ×, read *multiplied by*. Thus, 5 × 4 is read, 5 multiplied by 4.

Repeat the column once 1. 2 times 1. 3 times 1. 4 times 1. 5 times 1. 6 times 1. 7 times 1. 8 times 1. 9 times. 1. 10 times 1. 11 times 1. 12 times 1. What is the Sign of Multiplication?

Exercises on the Table.

1. 3 times 8 are how many?
2. 2 times 7 are how many?
3. 4 times 2 are how many?
4. 5 times 3 are how many?
5. 4 times 5 are how many?
6. 7 times 3 are how many?
7. 6 times 7 are how many?
8. 6 times 2 are how many?
9. 0 times 9 are how many?
10. 8 times 3 are how many?
11. 9 times 7 are how many?
12. 7 times 0 are how many?
13. 5 times 8 are how many?
14. 6 times 5 are how many?

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15. 8 × 9 = how many?
16. 10 × 2 = how many?
17. 9 × 0 = how many?
18. 7 × 2 = how many?
19. 8 × 4 = how many?
20. 11 × 3 = how many?
21. 12 × 5 = how many?
22. 3 × 9 = how many?
23. 12 × 2 = how many?
24 4 × 7 = how many?
25. 11 × 5 = how many?
26. 9 × 6 = how many?
27. 10 × 3 = how many?
28. 7 × 12 = how many?

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29. 6 times 11 are how many?
30. 7 times 7 are how many?
31. 5 times 10 are how many?
32. 4 times 9 are how many?
33. 11 times 10 are how many?
34. 8 times 8 are how many?
35. 9 times 9 are how many?
36. 11 times 11 are how many?
37 10 times 10 are how many?
38. 12 times 12 are how many?

Mental Exercises.

1. What will 3 oranges cost at 6 cents apiece?

SOLUTION. *Since* 1 *orange cost* 6 *cents,* 3 *oranges will cost* 3 *times* 6 *cents, which are* 18 *cents. Therefore,* 3 *oranges at* 6 *cents apiece will cost* 18 *cents.*

2. If a boat will carry 7 men, how many men will 4 boats of the same size carry?

3. If a quart of berries can be bought for 8 cents, *how many* cents will be required to buy 6 quarts?

4. How much can be obtained for 8 melons, at 12 cents apiece?

5. By reading 9 pages a day, how many pages can you read in 9 days?

6. What cost 12 tons of coal, at 5 dollars a ton?

7. What cost 11 pencils, at 10 cents apiece?

8. If a boat sails at the rate of 9 miles an hour, how far will it sail in 12 hours?

9. When flour is 8 dollars a barrel, how much must be paid for 7 barrels?

10. How many are 4 times 24?

SOLUTION. *As many as the product of* 24 *by* 4. 24 *is* 2 *tens and* 4 *units;* 4 *times* 2 *tens are* 8 *tens, or* 80 *units, and* 4 *times* 4 *units are* 16 *units; which added to* 80 *units equal* 96 *units. Therefore,* 4 *times* 24 *are* 96.

11. How many are 3 times 16? 3 times 35?

12. How many are 5 times 18? 4 times 27?

13. What cost 7 pounds of butter, at 41 cents a pound?

14. If 1 book costs 33 cents, how much will 5 books cost?

Written Exercises.

57. Let it be required to multiply 564 by 7.

OPERATION.

Multiplicand.	564
Multiplier	7
Product,	3948

For convenience, we write the multiplier under the units in the multiplicand, and begin with units to multiply.

7 times 4 units are 28 units, which equal 2 tens and 8 units; we write the 8 units, and reserve the 2 tens to add to

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Why is it convenient to begin with units to multiply? *Ans.* Because, if the product of any of the orders contains units of the next higher order, they may be readily added to the product of that order.

the next product. 7 times 6 tens are 42 tens, which with the 2 tens added are 44 tens, or 4 hundreds and 4 tens, we write the 4 tens, and reserve the 4 hundreds to add to the next product. 7 times 5 hundreds are 35 hundreds, which with the 4 hundreds added are 39 hundreds, or 3 thousands and 9 hundreds; which we write. Therefore, the product is three thousand nine hundred and forty-eight, or 3948.

2. Let it be required to multiply 736 by 206.

OPERATION.

Multiplicand,	736
Multiplier,	206
Partial Products,	4416
	14720
Product,	151616

For convenience, we write the multiplier under the multiplicand, so that figures of the same order stand in the same column; and multiplying by the units, as inthe preceding operation, we obtain 4416 units. There being 0 tens, we write a cipher underneath, and pass to the hundreds' figure of the multiplier. 2 hundreds are two hundred units, and 2 hundred times 6 units are 12 hundreds, or 1 thousand and 2 hundreds. We write the 2 hundreds, and reserve the 1 thousand to add to the next product. 2 hundred times 3 tens are 6 hundred tens, equal 6 thousands, and the 1 thousand added are 7 thousands, which we write. 2 hundred times 7 hundreds are 14 hundred-hundreds, equal 1 hundred-thousand, and 4 ten-thousands, which we write, and obtain 1472 hundreds, or, with the cipher on the right, 14720 tens. Adding the two partial products we have for the entire product 151616. Therefore, the product of 736 by 206 is 151616.

In practice, the name of the order of units may be omitted. Thus, in the operation of the first question: 7 times 4 are 28; we write the 8, and add the 2 to the next product; 7 times 6 are 42, and 2 are 44; we write 4, and add 4 to the next product; 7 times 5 are 35, and 4 are 39; answer, 3948.

Explain the operations.

Rule.

Write the multiplier under the multiplicand, so that units may stand under units, tens under tens, etc.

If the multiplier contains but one order, begin with units and multiply each order of the multiplicand by it, writing the right-hand figure of each product underneath, adding the other orders, if any, to the next product, observing to write all the figures of the last product.

If the multiplier contains more than one order, multiply by each order separately, writing the right-hand figure of each partial product under its own multiplier. The sum of the partial products will be the entire product.

Proof. Multiply the multiplier by the multiplicand; and, if the work is right, the product will be the same as that first obtained.

Examples.

3. Find the product of 534 by 242.

$$\begin{array}{rrcl} & 534 & & \\ & 242 & & \\ \cline{2-2} & 1068 & = & 534 \times 2 \\ & 2136 & = & 534 \times 40 \\ & 1068 & = & 534 \times 200 \\ \cline{2-2} \text{Ans.} & 129228 & = & 534 \times 242 \end{array} \qquad \text{Proof.} \left\{ \begin{array}{r} 242 \\ 534 \\ \hline 968 \\ 726 \\ 1210 \\ \hline 129228 \end{array} \right.$$

How do you multiply when the multiplier has but one figure? How when it has more than one figure? Why is the right-hand figure of each partial product written under its multiplier? *Ans.* That units of the same order may stand in the same column, for convenience in adding. Repeat the Rule. What is the Proof?

	(4.)	(5.)	(6.)	(7.)
Multiply	361	723	892	904
By	5	8	9	4
Ans.	1805	5784	8028	3616

(8.)	(9.)	(10.)	(11.)	(12.)
707	335	806	139	678
7	3	6	2	8

(13.)	(14.)	(15.)	(16.)
1031	2461	3008	1213
11	5	4	12

(17.)	(18.)	(19.)	(20.)
317	1426	1260	2371
13	15	14	23

(21.)	(22.)	(23.)	(24.)
1617	1212	8023	1441
121	192	66	315

25. Multiply 956 by 12. Ans. 11472.
26. Multiply 527 by 7. Ans. 3689.
27. Multiply 365 by 25. Ans. 9125.
28. Multiply 1836 by 32. Ans. 58752.

Why is a partial product of the same order of units as the figure of the multiplier which produced it? *Ans.* Because units multiplied by *units* produces *units;* units multiplied by *tens* produces *tens;* units multiplied by *hundreds* produces *hundreds*, etc.

29. Multiply 895 by 11. Ans. 9845.
30. 3706 × 207 = how many? Ans. 767142.
31. 2135 × 19 = how many? Ans. 40565.
32. 958 × 34 = how many? Ans. 32572.
33. 47696 × 144 = how many? Ans. 6868224.

58. Since the product of any number by 0 is 0, observe to write in an operation a cipher under each cipher of the multiplier.

If, however, the multiplier is 10, 100, 1000, etc., the product is obtained, at once, by writing after the multiplicand as many ciphers as are in the multiplier.

For, the value expressed by figures is made tenfold by each removal of them an order to the left. (Art. 33.) Thus 2 × 10 = 20, 2 × 100 = 200, etc.

	(34.)	(35.)	(36.)	(37.)
Multiply	519	785	162	2400
By	20	60	103	200
Ans.	10380			480000

38. Multiply 23467 by 10. Ans. 234670.
39. Multiply 68920 by 100.
40. Multiply 48005 by 3040. Ans. 145935200.

41. How much will a farm of 149 acres cost, at 96 dollars an acre? Ans. 14304 dollars.

42. If a ship sail 192 miles a day, how far will it sail in 56 days? Ans. 10752 miles.

Why do you write a cipher in the operation under each cipher of the multiplier? How do you multiply by 10, 100, 1000, etc.? Give the reason.

43. If a coach-wheel turn round 346 times in going a mile, how many times will it turn round in going 95 miles? Ans. 32870 times.

44. What will 49 yards of drilling cost, at 37 cents per yard? Ans. 1813 cents.

45. What will 145 tons of hay come to, at 15 dollars a ton? Ans. 2175 dollars.

46. How much will 320 barrels of flour cost, at 9 dollars a barrel? Ans. 2880 dollars.

47. How many men are there in an army of 20 regiments, of 980 men each? Ans. 19600.

48. What is the value of 130 United States bonds, at 113 dollars each? Ans. 14690 dollars.

49. If the average number of beats of the heart in childhood is 105 in a minute, what will be the number per hour, or 60 minutes? Ans. 6300.

50. If the earth in its orbit around the sun moves at the rate of 68000 miles an hour, how far does it move in one day or 24 hours? Ans. 1632000 miles.

Examples

COMBINING ADDITION OR SUBTRACTION AND MULTIPLICATION.

1. If 37 trees yield 13 bushels of apples each, and 41 others yield 11 bushels each, how many bushels do they all yield?

SOLUTION. *They all yield as many bushels as the products of* 37 *times* 13 *bushels, and* 41 *times* 11 *bushels;* 37 *times* 13 *bushels are* 481 *bushels, and* 41 *times* 11 *bushels are* 451 *bushels; and* 481 *bushels and* 451 *bushels are* 932 *bushels. Therefore, they all yield* 932 *bushels.*

REVIEW QUESTIONS.—What is Addition? (39.) Subtraction? *(43.) What is the* result of Addition called? (39.) What is the *result of Subtraction* called? (43.)

2. Bought 713 tons of coal for 2852 dollars, and sold it at 6 dollars a ton; how much was gained by the operation? Ans. 1426 dollars.

3. Sold 325 barrels of apples at 3 dollars a barrel, and received 650 dollars in cash, and a note for the balance. For what sum was the note?

4. A farmer bought 32 sheep at 6 dollars each, 31 cows at 45 dollars each, and 3 horses at 162 dollars each. He sold the whole for 2500 dollars; how much did he gain? Ans. 427 dollars.

5. A merchant bought 7 rolls of carpeting, each containing 27 yards, and 4 rolls each containing 18 yards. How many yards did he buy in all?
Ans. 261 yards.

6. If a man earn 950 dollars a year, and his expenses are 2 dollars a day, how much can he save in a year, or 365 days?

7. Jones and Smith start from the same place and travel in opposite directions, Jones at the rate of 32 miles a day, and Smith at the rate of 34 miles a day. How far apart will they be at the end of 6 days?
Ans. 396 miles.

8. Arthur has 135 dollars and his brother has 4 times as much. lacking 15 dollars; how much has his brother? Ans. 525 dollars.

9. Bought a dictionary for 6 dollars, a set of histories for 22 dollars, and a select library for 137 dollars; and gave in payment a check for 63 dollars, and three 50-dollar bills. How much should be received back? Ans. 48 dollars.

REVIEW QUESTIONS. — What is the larger of the two given numbers called in Subtraction? *Ans.* The Minuend. In Multiplication, what is the result called? (51.)

10. Two steamers start at the same time to meet each other, the one from Boston and the other from Liverpool, which places are 2883 miles apart. Provided that one sails at the rate of 192 miles a day, and the other at the rate of 168 miles a day, how many miles will they be from each other at the end of 5 days? Ans. 1083 miles.

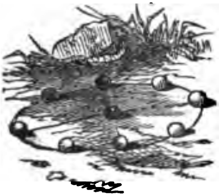
◆

DIVISION.

59. 1. At 2 cents each, how many apples can be bought for 8 cents?

Solution. *As many apples as* 2 *cents are contained times in* 8 *cents;* 2 *cents are contained in* 8 *cents,* 4 *times. Therefore, at* 2 *cents each,* 4 *apples can be bought for* 8 *cents.*

2. If 9 marbles be divided equally among 3 boys, how many marbles will each boy have?



Solution. *Each boy will have* 1 *of the* 3 *equal parts of* 9 *marbles;* 1 *of the* 3 *equal parts of* 9 *marbles is* 3 *marbles. Therefore, each boy will have* 3 *marbles.*

Here, in the first solution the process consists in finding how often one of two given numbers of the same name or kind is contained in the other; and, in the second, it consists in separating one of two

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Review Questions. — What is Multiplication? (50.) The Product? (51.) The Multiplicand? (52.) The Multiplier? (53.)

given numbers of different names or kinds into as many equal parts as there are units in the other. Both illustrate what is called DIVISION. Hence,

60. Division is the process of finding how many times one number is contained in another; or of finding one of the equal parts of a number.

One of *two* equal parts of a number is called *one half*. Thus, one half of two is one.

One of *three* equal parts of a number is called *one third*. Thus, one third of six is two.

One of *four* equal parts of a number is called *one fourth*. Thus, one fourth of four is one.

In like manner, one of the five equal parts of a number is called *one fifth*; one of the six equal parts is called *one sixth*; one of the ten equal parts is called *one tenth*; one of the one hundred equal parts is called *one hundredth*, etc.

Hence, a *whole* number has two halves, three thirds, four fourths, five fifths, ten tenths, one hundred hundredths, etc.

61. One or more equal parts of a unit are called *fractions*, to distinguish them from unbroken, or whole numbers, which are called *integers*, or *integral numbers*.

62. The DIVIDEND is the number to be divided.

63. The DIVISOR is the number by which we divide.

64. The QUOTIENT is the result or number obtained by the division.

Of what does the process in the second solution consist? What do both illustrate? What is Division? What is one of the two equal parts of a number called? One of three equal parts? One of four equal parts? One of five? One of six? What is the Dividend? The Divisor? The Quotient?

65. The **Remainder** is that part of the dividend which is left over after finding the exact whole number of the quotient. Thus, 3 is contained in 7, 2 times and 1 over as a remainder. The division of this 1, indicated by writing the divisor 3 underneath it, with a line between, gives $\frac{1}{3}$, which written after the whole number of the quotient, makes the complete quotient $2\frac{1}{3}$, read *two and one-third.*

Division Table.

1 in	2 in	3 in	4 in	5 in
1, 1 time	2, 1 time	3, 1 time	4, 1 time	5, 1 time
2, 2 times	4, 2 times	6, 2 times	8, 2 times	10, 2 times
3, 3 "	6, 3 "	9, 3 "	12, 3 "	15, 3 "
4, 4 "	8, 4 "	12, 4 "	16, 4 "	20, 4 "
5, 5 "	10, 5 "	15, 5 "	20, 5 "	25, 5 "
6, 6 "	12, 6 "	18, 6 "	24, 6 "	30, 6 "
7, 7 "	14, 7 "	21, 7 "	28, 7 "	35, 7 "
8, 8 "	16, 8 "	24, 8 "	32, 8 "	40, 8 "
9, 9 "	18, 9 "	27, 9 "	36, 9 "	45, 9 "
10, 10 "	20, 10 "	30, 10 "	40, 10 "	50, 10 "

6 in	7 in	8 in	9 in	10 in
6, 1 time	7, 1 time	8, 1 time	9, 1 time	10, 1 time
12, 2 times	14, 2 times	16, 2 times	18, 2 times	20, 2 times
18, 3 "	21, 3 "	24, 3 "	27, 3 "	30, 3 "
24, 4 "	28, 4 "	32, 4 "	36, 4 "	40, 4 "
30, 5 "	35, 5 "	40, 5 "	45, 5 "	50, 5 "
36, 6 "	42, 6 "	48, 6 "	54, 6 "	60, 6 "
42, 7 "	49, 7 "	56, 7 "	63, 7 "	70, 7 "
48, 8 "	56, 8 "	64, 8 "	72, 8 "	80, 8 "
54, 9 "	63, 9 "	72, 9 "	81, 9 "	90, 9 "
60, 10 "	70, 10 "	80, 10 "	90, 10 "	100, 10 "

What is the Remainder? Repeat the column 1 in 1. 2 in 2. 3 *in 3. 4 in 4.* 5 in 5. 6 in 6. 7 in 7. 8 in 8. 9 in 9. 10 in 10.

66. The SIGN OF DIVISION is a short horizontal line with a dot above and below it, ÷, read, divided by. Thus, 6 ÷ 2 is read, six divided by 2.

Sometimes, in place of the dots, the number divided is written above the line, and the number which divides it is written below. Thus, $\frac{6}{2}$ is read, six divided by two, or six halves.

Exercises on the Table.

1. 4 in 12 how many times?
2. 5 in 10 how many times?
3. 3 in 27 how many times?
4. 8 in 64 how many times?
5. 2 in 16 how many times?
6. 6 in 42 how many times?
7. 9 in 18 how many times?
8. 3 in 18 how many times?
9. 7 in 49 how many times?
10. 4 in 16 how many times?
11. 5 in 50 how many times?
12. 8 in 64 how many times?
13. 9 in 54 how many times?
14. 10 in 70 how many times?

15. 8 ÷ 2 = how many?
16. 18 ÷ 6 = how many?
17. 27 ÷ 3 = how many?
18. 56 ÷ 7 = how many?
19. 20 ÷ 2 = how many?
20. 36 ÷ 6 = how many?
21. 81 ÷ 9 = how many?
22. 30 ÷ 3 = how many?
23. 15 ÷ 5 = how many?
24. 24 ÷ 8 = how many?
25. 45 ÷ 9 = how many?
26. 40 ÷ 5 = how many?
27. 63 ÷ 7 = how many?
28. 72 ÷ 8 = how many?

29. 3 in 30 how many times?
30. 5 in 35 how many times?
31. 7 in 28 how many times?
32. 8 in 16 how many times?
33. 7 in 14 how many times?
34. 9 in 63 how many times?
35. 10 in 80 how many times?
36. 2 in 18 how many times?
37. 6 in 54 how many times?
38. 4 in 36 how many times?
39. 10 in 90 how many times?
40. 8 in 80 how many times?
41. 9 in 72 how many times?
42. 10 in 100 how many times?

What is the sign of Division? What sometimes takes the place of the dots?

Mental Exercises.

1. How many oranges, at 5 cents each, can be bought for 45 cents?

SOLUTION. *As many oranges as* 5 *cents are contained times in* 45 *cents, which are* 9. *Therefore,* 9 *oranges can be bought for* 45 *cents.*

2. If hats are 4 dollars each, how many can be bought for 24 dollars?
3. How many quarts of milk, at 6 cents a quart, can be bought for 42 cents?
4. When berries are 10 cents a quart, how many quarts can be bought for 60 cents?
5. At 7 dollars a ton, how many tons of coal can be bought for 49 dollars?
6. How many pounds of sugar, at 9 cents a pound, can be bought for 72 cents?
7. If you can travel 4 miles in an hour, how many hours will it take you to travel 36 miles?
8. When 3 barrels of flour cost 27 dollars, what will be the cost of 1 barrel?

SOLUTION. *The cost of* 1 *barrel will be one third of the cost of* 3 *barrels; and one third of* 27 *dollars is* 9 *dollars. Therefore, the cost of* 1 *barrel will be* 9 *dollars.*

9. If 7 men cut 42 cords of wood in a week, how many cords does 1 man cut?

REVIEW QUESTIONS.—When a number is divided into two equal parts, what is one of the parts called? When divided into three *equal parts*?

10. If 10 pounds of rice cost 90 cents, how much is it a pound?

11. A farmer sold 8 sheep for 40 dollars; how much apiece did he get for them?

12. At 30 dollars for 3 plows, what is the cost of each plow?

13. Mr. Rollins bought 9 barrels of flour for 72 dollars; what did he pay per barrel?

14. How many times is 8 contained in 96?

SOLUTION. 96 *is* 9 *tens and* 6; 8 *in* 9 *tens,* 1 *ten time, and* 1 *ten remaining;* 1 *ten remaining equals* 10 *units;* 10 *units and* 6 *units are* 16 *units; and* 8 *in* 16 *units* 2 *units times. Therefore,* 8 *is contained in* 96, 1 *ten and* 2 *units times, or* 12 *times.*

15. How many times is 5 contained in 65.

16. Divide 99 by 3. 88 by 4. 77 by 7.

17. How many times 5 in 125? In 175?

18. How many times 6 in 132? In 108?

19. If 9 writing-books cost 108 cents, what is the cost of each?

20. At 104 dollars for 8 tons of coal, what is the cost per ton?

67. Division is the *reverse* of multiplication. The *dividend* answering to the *product,* and the *divisor* and *quotient* to the *factors.*

When the division is performed mentally, except writing the quotient figures, the process is called SHORT DIVISION; and when the operation is written in full, the process is called LONG DIVISION.

Of what is Division the reverse? What is Short Division? Long Division? What does prefixed mean? Ans. Placed before

Written Exercises.

68. To divide by Short Division.

1. Let it be required to divide 1701 by 4.

OPERATION.

Divisor, 4)1701 Dividend.
425¼ Quotient.

For convenience we write the divisor at the left of the dividend, and begin at the left to divide.

4 is contained in 17 hundreds 4 hundreds times and 1 hundred remaining. We write the 4 hundreds. The 1 hundred we consider prefixed to the 0 tens, making 10 tens. 4 in 10 tens, 2 tens times, and 2 tens remaining. We write the 2 tens. The remaining 2 tens we consider prefixed to the 1 unit, making 21 units. 4 in 21 units, 5 units times, and 1 unit remaining. We write the 5 units. We then indicate the division of the remainder, 1 unit, by the divisor 4, and annex to the integral part of the quotient; and have for the complete quotient 425¼. Therefore, the quotient of 1701 divided by 4 is 425¼.

2. Let it be required to divide 763 by 7.

OPERATION.

Divisor, 7)763 Dividend.
109 Quotient.

For convenience we write the divisor and begin to divide as in the preceding operation.

7 in 7 hundreds, 1 hundred time, which we write. 7 in 6 tens, 0 tens times, which we write. We prefix the undivided 6 tens to the 3 units, making 63 units. 7 in 63 units, 9 times, which we write. Therefore, the quotient of 763 divided by 7 is 109.

In practice we may say, 7 in 7, 1, which we write 7 in 6, 0, which we write; prefix the 6 to the figure 3; 7 in 63, 9, which we write; answer, 109.

Review Questions. — What is the Proof in Addition? (41.) In Subtraction? (48.) In Multiplication? (57.)

Rule.

Write the divisor at the left of the dividend.

Begin at the left; divide the number expressed by the fewest figures of the dividend that will contain the divisor, and write the result beneath.

If there is a remainder, regard it as prefixed to the figure of the next lower order; divide as before, and so continue till all the figures of the dividend have been used.

If any partial dividend is less than the divisor, write a cipher in the quotient, and prefix the partial dividend to the figure of the next lower order, if any, for a new dividend.

If there be a final remainder, write it, with the divisor beneath, after the integral part of the quotient.

Proof. Multiply the whole number of the quotient by the divisor, and to the product add the remainder, if any; and the result will equal the dividend, if the work is right.

Examples.

	(3.)		(4.)	(5.)
Divisor,	5)1805	Dividend.	3)694	9)1890
	361	Quotient.	Ans. $231\frac{1}{3}$	Ans. 210
	5		3	9
Proof,	1805		Proof, 694	Proof, 1890

How do you write the divisor? Where do you begin to divide? How do you proceed if there is a remainder? How do you proceed when any partial dividend is less than the divisor? When there is a remainder after dividing the last figure? What is the Rule? The Proof?

(6.)	(7.)	(8.)	(9.)
4)8126	7)3514	9)3464	3)1005
$2031\frac{2}{4}$	502	$384\frac{8}{9}$	335

(10.)	(11.)	(12.)	(13.)
7)813	6)1705	9)9100	4)3787

(14.)	(15.)	(16.)	(17.)
7)80971	3)5280	8)90374	9)91080

18. Divide 57848 by 8. Ans. 7231.
19. Divide 8028 by 9. Ans. 892.
20. Divide 3616 by 4. Ans. 904.
21. Divide 2275 by 7. Ans. 325.
22. $50405 \div 5 =$ how many? Ans. 10081.
23. $65412 \div 3 =$ how many? Ans. 21804.
24. $947 \div 4 =$ how many? Ans. $236\frac{3}{4}$.
25. $\frac{54636}{3} =$ how many? Ans. 18212.

69. When the divisor is 10, 100, 1000, etc., the quotient is obtained, at once, by cutting off at the right of the dividend as many figures as there are ciphers in the divisor.

For, since figures are multiplied by 10 by annexing one cipher, by 100 by annexing two ciphers, etc., (Art. 58,) and as division is the reverse of multiplication, cutting off one figure at the right of the dividend divides it by 10, cutting off two figures divides it by 100, etc.

When the divisor is 10, 100, 1000, etc., how can the quotient be obtained at once? How are figures multiplied by 10? By 100? *How can* you divide by 10? By 100?

A dot (.) called the *Decimal Point,* may be used to denote where the cutting off or separation is made, and the remainder will be the part on the right of the point. Thus, 1326 ÷ 10 = 132.6 = $132\frac{6}{10}$, read, one hundred thirty-two units and six tenths; 1326 ÷ 100 = 13.26 = $13\frac{26}{100}$, read, thirteen units and twenty-six hundredths; 1326 ÷ 1000 = 1.326 = $1\frac{326}{1000}$, read, one unit and three hundred twenty-six thousandths; etc. That is,

The first order at the right of the decimal point expresses tenths; the second, hundredths; the third, thousandths, etc.

26. Divide 1975 by 10. Ans. $197\frac{5}{10}$.
27. Divide 4576 by 100. Ans. $45\frac{76}{100}$.
28. Divide 901 by 100. Ans. $9\frac{1}{100}$.
29. Divide 1860 by 10. Ans. 186.
30. Divide 17930 by 1000. Ans. $17\frac{930}{1000}$.

31. At 6 dollars each, how many sheep can be purchased for 1806 dollars? Ans. 301.

32. At 10 dollars a barrel, how many barrels of flour can be purchased for 1440 dollars? Ans. 144.

33. 7 men working together earned 1547 dollars; what is each man's share of it? Ans. 221 dollars.

34. A gentleman dying left 24560 dollars, to be divided equally between his wife and three sons. What will be the share of each? Ans. 6140 dollars.

35. John Stevens sold melons at 9 cents each, to the amount of 2943 cents. How many did he sell?

How do we denote where the cutting-off is made? Why is the decimal point so called? *Ans.* From *decem,* the Latin for ten. What does the first order at the right of the decimal point express? The second order? The third?

36. How many watches, at 100 dollars each, can be bought for 978000 dollars ?

37. If a man can do a piece of work in 365 days, how many days will it take 5 men to do the same?
Ans. 73.

38. If a prize amounting to 95163 dollars be divided between 8 men, how much will each receive ?
Ans. 11895$\frac{3}{8}$ dollars.

39. At 6 dollars a ton, how much coal can be bought for 19201 dollars ? Ans. 3200$\frac{1}{6}$ tons.

70. To divide by Long Division.

1. Let it be required to divide 34531 by 15.

OPERATION.

Dividend.
Divisor 15)34531(2302$\frac{1}{15}$ Quotient.
30
—
45
45
—
031
30
—
1 Remainder.

We write the divisor, and begin to divide as in short division.

15 is contained in 34 thousands, 2 thousand times; we write the 2 thousand in the quotient, at the right of the dividend. 15 × 2 thousand = 30 thousand, which subtracted from 34 thousand, leave 4 thousand. Bringing down the figure of the next lower order of the dividend, we have

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Review Questions. — When is Division called Short Division ? (67.) When called Long Division ? (67.) How do Short and Long Division differ ? *Ans.* In the former the answer only is written, but *in the latter* the work as well as the answer is written.

45 hundred, in which 15 is contained 3 hundred times; we write the 3 hundred in the quotient. 15 × 3 hundred = 45 hundred, which subtracted from 45 hundred, leaves nothing. Bringing down the next figure, we have 3 tens, in which 15 is contained 0 tens times; we write the 0 tens in the quotient, and bringing down the next figure, we have 31 units, in which 15 is contained 2 times; we write the 2 in the quotient. Multiplying and subtracting, as before, we have a remainder of 1; and, for the complete quotient, $2302\frac{1}{15}$.

In practice, we may say: 15 in 34, 2 times; write 2 in the quotient; 15 × 2 = 30, which from 34 leaves 4. Bring down 5; 15 in 45, 3 times; write 3 in the quotient; 15 × 3 = 45, which from 45 leaves 0. Bring down 3; 15 in 3, 0 times; write 0 in the quotient. Bring down 1; 15 in 31, 2 times; 15 × 2 = 30, which from 31 leaves 1. Answer, $2302\frac{1}{15}$.

Rule.

Write the divisor at the left of the dividend.

Begin at the left; divide the number expressed by the fewest figures of the dividend that will contain the divisor, and write the quotient at the right of the dividend.

Multiply the divisor by this quotient; subtract the product from the part of the dividend used, and to the remainder bring down the next figure of the dividend.

Divide as before, and so continue till all the figures of the dividend have been used.

If there be a final remainder write it, with the divisor beneath, after the integral part of the quotient.

Proof. The same as in short division.

How do you write the divisor? Where do you begin to divide? How do you multiply and subtract? What is the Rule? The Proof?

Examples.

2. Divide 26639 by 19.

OPERATION.	PROOF.
19)26639(1402$\frac{1}{19}$, Ans.	1402
19	19
76	12618
76	1402
039	26638
38	1 Remainder.
1 Remainder.	26639

3. Divide 88763 by 32.

OPERATION.

32)88763(2773$\frac{27}{32}$
64
247
224
236
224
123
96
27

4. Divide 400000 by 96.

OPERATION.

96)400000(4166$\frac{64}{96}$
384
160
96
640
576
640
576
64

Review Questions.—What is an Operation? (13.) A Rule? *14.)* *What* an Example? (p. 20.) What is an Exercise? (p. 10.)

Divide,

5. 6780 by 15. Ans. 452.
6. 9125 by 25. Ans. 365.
7. 58752 by 32. Ans. 1836.
8. 40566 by 19. Ans. $2135\frac{1}{19}$.
9. 32572 by 34.
10. 29880 by 95.
11. 3762 by 24. Ans. $156\frac{18}{24}$.
12. 7168 by 28. Ans. 256.
13. 6750 by 15. Ans. 450.
14. 1748 by 18. Ans. $97\frac{2}{18}$.
15. 22200 by 222.
16. 9125 by 25. Ans. 365.
17. 45678 by 16. Ans. $2854\frac{14}{16}$.
18. 462442 by 183.
19. 5904 by 82. Ans. 72.
20. 208126 by 345. Ans. $603\frac{91}{345}$.
21. 65705 by 73. Ans. $900\frac{5}{73}$.
22. 14220 by 316. Ans. 45.
23. 2780 by 13. Ans. $213\frac{11}{13}$.
24. 42024 by 103. Ans. 408.
25. 27030 by 19.
26. 45817 by 35.
27. 6318 by 27. Ans. 234.
28. 77112 by 204. Ans. 378.
29. 9990 by 54. Ans. 185.
30. 34331 by 11. Ans. 3121.
31. 11520 by 12.
32. 66327 by 320. Ans. $207\frac{87}{320}$.
33. 46800 by 144. Ans. 325.
34. 241086 by 1132. Ans. $212\frac{1102}{1132}$.
35. 727604 by 3602. Ans. 202.
36. 437850 by 75. Ans. 5838.

37. If 630 dollars be divided equally among 15 men, how much will each receive? Ans. 42 dollars.

38. At the rate of 31 miles an hour, how long will a train of cars be in going 1116 miles? Ans. 36 hours.

Review Questions. — What is Division? (60.) What is the number to be divided called? (62.) The number by which we divide? (63.)

39. If a ship sail 3648 miles in 24 days, at what rate is that per day? Ans. 152 miles.

40. If 12 schools together have 1932 scholars, what is the average number of each?

Ans. 161 scholars.

41. When 16 acres cost 2304 dollars, what is the cost per acre? Ans. 144 dollars.

42. If 100 men share equally a prize of 17650 dollars, what is the share of each?

43. When 110 plows are worth 1320 dollars, what is each plow worth? Ans. 12 dollars.

44. If 21 bushels of wheat weigh 1260 pounds, what is the weight of one bushel?

45. 25 workmen received for a job 3155 dollars, to be equally divided among them; how much did each one receive? Ans. $126\frac{5}{25}$ dollars.

46. A field containing 11 acres produced 746 bushels of corn; what was the yield per acre?

Ans. $67\frac{9}{11}$ bushels.

47. If a person can copy 50 words in a minute, how long will he be in copying 47569 words?

Ans. $951\frac{19}{50}$ minutes.

48. At a sale, 28 carriages sold for 6972 dollars; what was the average rate? Ans. 249 dollars.

49. The salary of the President of the United States is 25000 dollars a year; how much is that a day, there being 365 days in a year? Ans. $68\frac{180}{365}$.

50. The distance from the earth to the moon is 240000 miles; how long would a person be in reaching it, supposing he could move regularly 30 miles an hour? Ans. 8000 hours.

Review Questions.—What is Notation? (32.) Numeration? (36.)

Exercises on the Preceding Rules.

1. Add together ninety thousand three hundred and forty, one thousand three hundred and six, and two.
Ans. 91648.

2. Subtract five thousand three hundred and seventeen, from sixteen thousand one hundred and thirty-four.

3. A and B start from the same point and travel in opposite directions. How far apart are they when A has gone 67 miles and B 139 miles?

4. The smaller of two numbers is 569, and their difference is 731; what is the larger number?
Ans. 1300.

5. A and B start from the same point and travel in the same direction. How far apart are they when A has gone 198 miles and B 319 miles?

6. The product of two factors is 4680, and one of the factors is 15; what is the other factor?
Ans. 312.

7. Bought goods to the amount of 3467 dollars, paying for them in corn 421 dollars, in wheat 968 dollars, and the rest in money. How much money was paid?

8. From $(612 + 12)$ take $(316 - 32)$. Ans. 340.

9. $(591 \times 3) + 29 =$ how many? Ans. 1802.

10. Gave 3375 dollars and 5 horses, worth each 125 dollars, for a farm of 160 acres; what was the cost of the farm per acre? Ans. 25 dollars.

11. What number is that which if added to 1000 will amount to 5417? Ans. 4417.

Review Questions.—What is Addition? (39.) Subtraction? (43.) Multiplication? (50.)

12. Bought 3 barrels of flour at 12 dollars a barrel, and 7 tons of hay at 20 dollars a ton, and paid for the whole 44 sheep. How much each was allowed for the sheep? Ans. 4 dollars.

13. (565 + 10) ÷ 25 = how many? Ans. 23.

14. Paid one-half of 36730 dollars for some oil lands, which turned out to be 963 dollars more than their true value; what were they worth?

Ans. 17402 dollars.

15. If the yearly income of a farmer is 2163 dollars, and from it he has to pay 122 dollars for repairs, 35 dollars for taxes, 516 dollars for hired help, and 805 dollars for support of his family, how much can he save? Ans. 685 dollars.

16. A drover has 2144 dollars, with which he wishes to buy horses at 165 dollars each, and then lay out the remainder for sheep at 4 dollars each. How many of each can he buy?

Ans. 12 horses, and 41 sheep.

◆

UNITED STATES MONEY.

71. United States Money is the legal currency of the United States.

What is United States Money?

Table

10 mills	make	1 cent,	marked	c.
10 cents,	"	1 dime,	"	d.
10 dimes,	"	1 dollar,	"	$
10 dollars,	"	1 eagle,	"	E.

72. The Coins of the United States are of bronze, nickel, silver, and gold, as follows :—

Bronze.	Cent,	value	1 c.
	Two-cent piece,	"	2 c.
Nickel.	Three-cent piece,	"	3 c.
Silver.	Three-cent piece,	"	3 c.
	Half-dime,	"	5 c.
	Dime,	"	10 c.
	Quarter-dollar,	"	25 c.
	Half-dollar,	"	50 c.
	Dollar,	"	$ 1.
Gold.	Dollar,	"	$ 1.
	Quarter-eagle,	"	$ 2½.
	Three-dollar piece,	"	$ 3.
	Half-eagle,	"	$ 5.
	Eagle,	"	$ 10.
	Double-eagle,	"	$ 20.

Repeat the Table. Of what are Coins of the United States? Which are of Bronze? Of Silver? Of Gold?

73. In accounts, eagles are expressed in dollars, dimes in cents, and mills often in parts of a cent.

Thus, 3 eagles are called 30 dollars; 2 dimes, 20 cents; and 5 mills, often, half a cent.

74. The Dollar is the *unit* of United States Money.

Eagles are *tens* of dollars, dimes are *tenths*, cents *hundredths*, and mills *thousandths*, of a dollar, and they are separated from dollars by the decimal point, (.)

Hence 44 eagles, 4 dollars, 4 dimes, 4 cents, and 4 mills, are written

$444.444,

and read four hundred forty-four dollars, forty-four cents, and four mills.

75. In United States Money, 10 of a lower denomination make 1 of the next higher, according to the principle of the Arabic method of notation (33). Hence,

United States Money may be written, added, subtracted, multiplied, and divided, by preceding rules.

How are Eagles expressed in accounts? Dimes? Mills, often? What is the unit of United States Money? What are Eagles? Dimes? Cents? Mills? How are dimes, cents, and mills separated from dollars? How many of a lower denomination in United States Money make 1 of the next higher? How may United States Money be written, added, subtracted, etc.?

Mental Exercises.

1. How many mills are there in 7 cents?

Solution. *Since in* 1 *cent there are* 10 *mills, in* 7 *cents there are* 7 *times* 10 *mills, which are* 70 *mills. Therefore, in* 7 *cents there are* 70 *mills.*

2. How many cents are there in 8 dimes?
3. How many dollars are there in 5 eagles?
4. How many dimes are there in 6 dollars?
5. How many dollars are there in 4 half-eagles?
6. How many cents are there in six half-dimes?
7. How many cents are there in 4 quarter-dollars?
8. How many dollars are there in 3 double-eagles?
9. How many cents are there in 70 mills?

Solution. *Since in* 10 *mills there is* 1 *cent, in* 70 *mills there are as many cents as* 10 *mills are contained times in* 70 *mills, which are* 7. *Therefore, there are* 7 *cents in* 70 *mills.*

10. How many dimes are there in 60 cents?
11. How many dollars are there in 100 cents?
12. How many dollars are there in 300 cents?
13. How many dollars are there in 70 dimes?

Written Exercises.

Copy and read the following: —

1. $ 7.65.
2. $ 12.20.
3. $ 37.02.
4. $ 16.125.
5. $ 77.005.
6. $ 11.111.
7. $ 100.06.
8. $ 363.55.
9. $ 217.755.

Explain the first solution. The second.

Write in figures : —

10. Twenty-five dollars, fifteen cents. Ans. $ 25.15.
11. Sixty-three dollars, seven cents. Ans. $ 63.07.
12. One hundred dollars, ten cents. Ans. $ 100.10.
13. Two dollars, two cents, and five mills. Ans. $ 2.025.
14. No dollars and sixteen cents. Ans. $ 0.16.
15. Twenty-five dollars and five mills. Ans. $ 25.005.

ADDITION.

76. 1. Add $ 21.38, $ 1.82, and $ 31.015.

OPERATION

$ 21.38
1.82
31.015
$ 54.215

For convenience, we write the numbers so that figures of the same order shall stand in the same column, and begin at the right to add, observing to place the decimal point between dollars and cents.

(2.)	(3.)	(4.)	(5.)
$ 100.31	$ 91.636	$ 167.345	$ 18.45
19.435	160.05	70.00	29.68
3.005	10.109	707.07	106.05
Ans. $ 122.750			$ 154.18

6. What is the sum of $ 960.17, $ 817.05, $ 3.09, and $29.19 ?

7. Bought a coat for $ 12.50, a vest for $ 6, a pair of shoes for $ 2.12½, and a handkerchief for $ 0.87½ ; how much did the whole cost ? Ans. $ 21.50.

How are the numbers written in Addition of United States Money? Why? *Ans.* Because only numbers of the same denomination can be added? Where do you begin to add? Where do you place the decimal point in the result? How many mills in half a cent?

8. Sold a horse for $208, a cow for $70, and some hay for $27.50; how much did the whole bring?

9. Albert Dayton owes A $145.91, B $1690.84, C $345.11, D $918.85, and E $1144.75; what is the whole amount of his debts? Ans. $4245.46.

10. If I give for a plow $10.63, for a wagon $86.75, for a load of hay $17.15, and for a hoe $1.25, how much do they all cost me?

11. A merchant owes three hundred and seven dollars, six hundred twenty-two dollars and eighty-eight cents, sixty-two dollars 17 cents and 5 mills, 325 dollars 12 cents and 5 mills; how much does he owe? Ans. $1317.18.

SUBTRACTION.

77. 1. Subtract $109.02½ from $963.03.

OPERATION.

$963.030
109.025
———
$854.005

For convenience, we write the subtrahend under the minuend, so that figures of the same order shall stand in the same column, and begin at the right to subtract, observing to place the decimal point between dollars and cents.

Here, in the subtrahend, we write the ½ cent as 5 mills; and in the minuend note the place of mills by 0.

(2.)	(3.)	(4.)	(5.)
$807.09	$420.35	$87.305	$8.340
191.63	319.06	19.32	6.765
$615.46			$1.575

6. Subtract $59.23 from $319.07. Ans. $259.84.

7. Subtract $87.62½ from $93.08. Ans. $5.455.

How do you write the numbers for subtracting? Where do you begin to subtract? Where place the decimal point?

8. On going to the village I had in my pocket $ 102.50; but I spent in a store $ 37.85. How much had I then left? Ans. $ 64.65.

9. From $ 7 take 7 cents. Ans. $ 6.93.

10. From $11 take 1 mill. Ans. $ 10.999.

11. Bought a slate for 25 cents, an arithmetic for 60 cents, some other books for $ 12.125, and some stationery for $1.62½, and gave in payment a twenty-dollar note. How much change should be received? Ans. $ 5.40,

MULTIPLICATION.

78. 1. Multiply $ 316.12 by 5.

OPERATION.

$$\begin{array}{r} \$\,316.12 \\ 5 \\ \hline \$\,1580.60 \end{array}$$

We begin at the right to multiply, observing to place the decimal point between dollars and cents in the product.

(2.)	(3.)	(4.)
$ 7.14	$ 210.06	$ 40.625
14	37	24
2856	147042	162500
714	63018	81250
$ 99.96	$ 7772.22	$975.000

5. Multiply $ 192.075 by 108. Ans. $ 20744.10.

6. Multiply $ 47.56 by 16.

7. What cost 9 tons of coal at $ 5.50 per ton? Ans. 49.50.

REVIEW QUESTIONS.—What is United States Money? What are coins? *Ans.* Pieces of metal converted into money by legal *stamping.* *Of what* metals are the coins of the United States?

8. What will be the cost of 563 bushels of wheat at $2.25 per bushel? Ans. $1266.75.

9. Bought 260 pounds of sugar at $0.125 per pound, and sold it at $0.15 per pound; what was the gain in the operation? Ans. $6.50.

10. Sold 4 cords of wood at $8 a cord, 3 tons of hay at $32.50 each, taking in part payment a cow worth $63 and the balance in money. How much money was received? Ans. $66.50.

11. What cost 36 pairs of shoes at $2.12½ each?

12. What cost 31 yards of broadcloth at $6.50 per yard? Ans. $201.50.

13. Sold 316 pairs of boots at $6.50 per pair, and received in payment $1000 down, and the balance in a note. For how much was the note? Ans. $1054.

DIVISION.

79. 1. Divide $65.56 by 8.

OPERATION.

8)$65.56

8.19\frac{4}{8}$

or,

8)$65.560

$8.195

Dividing the dollars and the cents, we have a remainder which we write as a part of a cent.

In the second operation, the division is extended to mills by supplying the mills' place in the dividend. The two answers, although a little different in form, express the same value.

How may the division be extended from dollars to cents, when there is a remainder, and no cents in the dividend? *Ans.* By supplying the two places of cents by ciphers. How may the division be extended to mills from cents, when there is a remainder, and no mills in the dividend? *Ans.* By supplying the place of mills by a cipher.

2. Divide $ 497 by 4.

OPERATION.

4)$ 497	or,	4)$ 497.00
$ 124¼		$ 124.25

(3.)	(4.)	(5.)	(6.)
9)$ 648.81	8)$ 108	6)$ 986.40	7)$ 684.614
72.09			$ 97.802

7. Divide $ 2764.25 by 7. Ans. $ 394.892+.

Here, in the answer, the sign + shows that the division is not exact.

8. Paid $ 78 for 12 sheep; how much was that for each? Ans. $ 6.50.

9. A merchant paid $ 255 for 75 sheep; how much was that a head? Ans. $ 3.40.

10. If $ 11.44 be paid for 26 yards of drilling, what is the cost per yard? Ans. $ .44.

11. If 24 men receive $ 637 to be equally divided among them, how much is each man's share? Ans. $ 26.541+.

12. Bought 25 yards of cambric for $ 4; how much was that a yard? Ans. $ .16.

13. A farmer expended $ 427.12½ for 201 sheep; for how much apiece must he sell them, so as neither to gain nor lose? Ans. $ 2.125.

What does the sign + written after an answer indicate?

BILLS AND ACCOUNTS.

80. A **Bill** is a written statement of merchandise bought or sold, or of services rendered.

81. An **Account** is a written statement of debt and credit, between two parties.

The party owing is the *Debtor*, and the party owed is the *Creditor*.

82. A bill is *receipted* when its payment is acknowledged by the party in whose favor it is, or by some one authorized to act for him.

Reckon and find the amount due on each of the following bills.

(1.) New York, May 16, 1865.

Mr. James Walsh,

Bought of J. B. Dalton & Co.,

16 Reams Paper,	at $7.25,	$
25 Slates,	" .40,	
20 Inkstands,	" .37,	
		$133.40

Received payment,

J. B. Dalton & Co.

What is a Bill? An Account? Which party is the Debtor? Which the Creditor? When is a bill receipted?

(2.) New Orleans, Sept. 9, 1865.

Mr. Louis Barrot,

To Felix Dumas & Co., Dr.

To 630 gal. Molasses, at $.60,
" 1963 lbs. Sugar, " .17,
" 3610 " Cotton, " .31,

$1830.81

◆

(3.) St. Louis, July 1, 1865.

Mr. John W. Curtis,

1865. To Henry D. Felt, Dr.

Jan. 8. To 365 bu. Wheat, at $2.20,
Feb. 7. " 162 " Corn, " .50,
June 9 " 192 " Barley, " .60,
" 12. " 20 bbl. Flour, " 8.50,

$1169.20

1865. Cr.

March 8. By Cash, $803.00
May 14. " Merchandise, 175.00
June 15. " 50 bags, at $.60, 30.00

$1008.00

Balance due H. D. F., $161.20

Received payment,
Henry D. Felt.

WEIGHTS AND MEASURES.

Avoirdupois Weight.

83. This weight is used for nearly all articles estimated by weight, except gold, silver, and jewels.

Table.

16	drams (dr.)	make	1 ounce,	oz.
16	ounces,		1 pound,	lb.
25	pounds,		1 quarter,	qr.
4	quarters,		1 hundred-weight,	cwt.
20	hundred weight,		1 ton,	T.

Formerly 112 pounds were reckoned a hundred-weight, and 2240 pounds a ton, but seldom now, except for coal at the mines, or goods at the United States custom-houses.

Mental Exercises.

1. How many drams in 2 ounces ? In 4 ounces ?
2. How many ounces in 3 pounds ? In 5 pounds ?
3. How many pounds in 3 quarters ? In 5 quarters ?
4. How many quarters in 5 hundred-weight ?
5. How many hundred-weight in 2 tons ? In 5 tons ?
6. How many ounces in 32 drams ? In 48 drams ?
7. How many pounds in 64 ounces ? In 48 ounces ?
8. How many pounds in 4 hundred-weight ? In 10 hundred-weight ?

For what is Avoirdupois Weight used ? Repeat the Table. How many pounds were formerly a hundred-weight ? A ton ?

9. How many hundred-weight in 8 quarters? In 24 quarters?

10. How many tons in 40 hundred-weight? In 80 hundred-weight?

Troy Weight.

84. This weight is used for gold, silver, and jewels.

Table.

24 grains (gr.)	make	1 pennyweight,	pwt.
20 pennyweights,		1 ounce,	oz.
12 ounces,		1 pound,	lb.

Apothecaries, in mixing medicines, use the *pound*, *ounce* (℥), and *grain*, of this weight; but divide the ounce into 8 *drams* (ʒ), each equal to three *scruples* (℈), each scruple being equal to 20 *grains*.

A pound Avoirdupois is equal to 7000 grains Troy.

Mental Exercises.

1. How many grains in 2 pennyweights? In 3 pennyweights?

2. How many pennyweights in 3 ounces? In 5 ounces?

3. How many ounces in 4 pounds? In 5 pounds?

4. How many pennyweights in 48 grains? In 72 grains?

5. How many pounds in 32 ounces? In 48 ounces?

For what is Troy Weight used? Repeat the Table? What denominations of Troy Weight do apothecaries use? How is the ounce divided by them? How many grains Troy are equal to a pound Avoirdupois?

Linear Measure.

85. Linear or Long Measure is used for distances and the length of articles.

Table.

12 inches (in.) make	1 foot,	ft.
3 feet,	1 yard,	yd.
5½ yards,	1 rod,	rd.
40 rods,	1 furlong,	fur.
8 furlongs,	1 mile,	m

Also, 4 inches make 1 hand; 6 feet 1 fathom; 3 miles 1 league.

In measuring *cloth* and other woven fabrics, the yard is divided into *halves*, *eighths*, and *sixteenths*, or *nails.*

Mental Exercises.

1. How many inches in 2 feet? In 7 feet?
2. How many feet in 4 yards? In 11 yards?
3. How many yards in 2 rods? In 4 rods?

Solution. *Since there are* 5½ *yards in* 1 *rod, there are* 2 *times* 5½ *yards, or* 2 *times* 5 *yards and* 2 *times* ½ *yard in* 2 *rods;* 2 *times* 5 *yards are* 10 *yards, and* 2 *times* ½ *a yard is* 1 *yard;* 10 *yards and* 1 *yard are* 11 *yards. Therefore, there are* 11 *yards in* 2 *rods*

4. How many rods in 2 furlongs? In 3 furlongs?
5. How many furlongs in 6 miles? In 9 miles?

For what is Linear Measure used? Repeat the Table. How many inches make 1 hand? How many feet 1 fathom? How many miles 1 league? How is the yard divided in measuring cloth?

6. How many yards in 18 feet? In 30 feet?

7. How many rods in 11 yards?

Solution. *Since there is in 5½ yards, or in 11 half yards, 1 rod, there are as many rods in 11 yards or 22 half yards, as 11 half yards are contained times in 22 half yards, which are 2. Therefore, there are 2 rods in 11 yards.*

8. How many furlongs in 80 rods? In 120 rods?

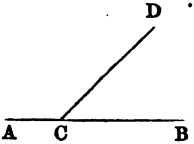
9. How many miles in 16 furlongs? In 24 furlongs?

Surface Measure.

86. A Surface is that which has only length and breadth.

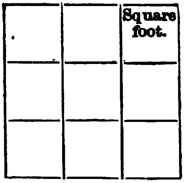
87. The Area of a figure is its quantity of surface.

88. An Angle is the difference in direction of two straight lines, which meet at a point. Thus, ACD and DCB are angles.



89. A Square is a figure having four equal sides and four equal angles.

The area of a square, each of whose sides is 1 yard, is 1 *square yard.*



1 Square Yard.

90. A Rectangle is any figure having four sides and four equal angles.

In the figure are 9 small squares, and if they are supposed to be 1 inch long and wide, the area will

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What is a Surface? The Area of a figure? An Angle? A Square? *A Rectangle?*

be 9 square inches. It is, therefore, evident that, by multiplying the number of units in the one side by those in the other, we obtain the number of small squares in the rectangle. Hence,

The area of a rectangle is equal to the product of the length by the breadth

91. Surface or Square Measure is used in measuring surfaces.

Table.

144 square inches (sq. in.) make	1 square foot,	sq. ft.
9 square feet,	1 square yard,	sq. yd.
30¼ square yards,	1 square rod or perch,	P.
40 square rods or perches,	1 rood,	R.
4 roods,	1 acre,	A.
640 acres,	1 square mile,	M.

Land is usually measured by Gunter's chain, which is 4 rods, or 66 feet, long, and divided into 100 links, each $7\frac{92}{100}$ inches long. 1 *square chain* is 16 square rods, and ten square chains are 1 acre. 640 acres make one section of land.

Mental Exercises.

1. How many square inches in 1 square foot?
2. How many square feet in 5 square yards?
3. How many square yards in 1 square rod? In 4 square rods?
4. How many square rods in 2 roods? In 3 roods?

How is the Area of a Rectangle found? For what is Surface Measure used? Repeat the Table. How is land usually measured?

5. How many roods in 5 acres? In 10 acres?

6. How many acres make 1 square mile?

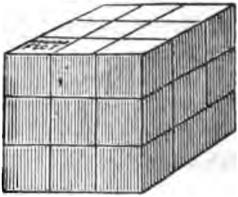
7. How many square yards in 45 square feet? In 81 square feet?

8. How many square rods in 121 square yards?

9. How many roods in 80 square rods? In 120 square rods?

10. How many acres in 16 roods? In 28 roods?

Solid Measure.

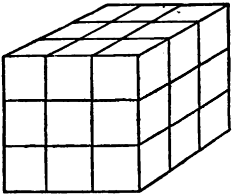

1 Solid Yard.

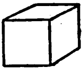
92. A Solid, or Body, is that which has length, breadth, and hight or depth.

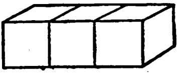
93. A Cube is a body bounded by six square and equal sides or faces.

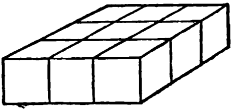
If a cube is 1 yard or 3 feet long, 1 yard or 3 feet wide, and 1 yard or 3 feet high, it is a cubic or solid yard.

94. A Rectangular Solid is a body bounded by rectangular faces.









In the large cube are represented 3 tiers of small cubes, of 3 rows each, and 3 in a row. If each of

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What is a Solid? A Cube? What is a Rectangular Solid?

these small cubes is supposed to be one solid foot, then in the large cube there evidently are in one of the equal tiers 3 times 3 solid feet, or 9 solid feet, and in the 3 tiers, 3 times 9 solid feet, or 27 solid feet, or as many as the product of the length, breadth, and hight or depth. Hence,

The contents of a rectangular solid is equal to the product of the length, breadth, and hight or depth.

95. Solid or Cubic Measure is used in measuring bodies, or things having length, breadth, and hight or depth.

Table.

1728	cubic inches (cu. in.) make	1 cubic foot,	cu. ft.
27	cubic feet,	1 cubic yard,	cu. yd.
40	cubic feet,	1 ton,	T.
128	cubic feet,	1 cord,	C.

A cord of wood is a pile 8 feet long, 4 feet wide, and 4 feet high. A cord foot (c. f.) is 1 foot in length of this pile, or 16 cubic feet.

1 Cord. 1 c. f.

A perch of masonry, or of building stone, is 24¾ cubic feet.

Mental Exercises.

1. How many cubic inches in 1 cubic foot?
2. How many cubic feet in 2 cubic yards?

What is the contents of a Rectangular Solid equal to? For what is Solid Measure used? Repeat the Table. What is a cord of wood? A cord foot? A perch of stone?

3. How many cubic feet in 2 tons? In 3 tons?
4. How many cubic feet in a cord?
5. How many cubic yards in 54 cubic feet? In 81 cubic feet?
6. How many tons in 80 cubic feet? In 120 cubic feet?

Liquid Measure.

96. Liquid Measure is used in measuring all kinds of liquids.

Table.

4 gills (gi.)	make	1 pint,	pt.
2 pints,		1 quart,	qt.
4 quarts,		1 gallon,	gal.
63 gallons,		1 hogshead,	hhd.

The liquid gallon contains 231 cubic inches. Formerly ale, beer, porter, and milk were sold by what was called *Beer Measure*, and of which the gallon contained 282 cubic inches.

In some States 31½ gallons, and in others, 28 to 32 gallons are considered a barrel.

Mental Exercises.

1. How many gills in 4 pints? In 8 pints?
2. How many pints in 8 quarts? In 10 quarts?
3. How many quarts in 5 gallons? In 9 gallons?
4. How many gallons in 1 hogshead?
5. How many pints in 16 gills? In 32 gills?
6. How many gallons in 20 quarts? In 36 quarts?
7. How many hogsheads in 63 gallons?

For what is Liquid Measure used? Repeat the Table. How many cubic inches in a liquid gallon? How were beer and milk formerly sold? *How many* gallons in a barrel?

Dry Measure.

97. Dry Measure is used in measuring such articles as grain, fruit, roots, coal, etc.

Table.

2	pints (pt.)	make	1 quart,	qt.
8	quarts,		1 peck,	pk.
4	pecks,		1 bushel,	bu
36	bushels,		1 chaldron.	ch.

A gallon, or 4 quarts, Dry Measure, contains 2 cubic inches, and a bushel, 2150$\frac{42}{100}$ cubic inches.

Mental Exercises.

1. How many pints in 8 quarts? In 10 qua
2. How many quarts in 4 pecks? In 6 pecks
3. How many pecks in 4 bushels? In 8 bush
4. How many bushels in 2 chaldrons?
5. How many quarts in 32 pints? In 40 pi
6. How many bushels in 16 pecks? In 32 pec
7. How many chaldrons in 72 bushels?

Time.

98. Time is a measured portion of duration.

Table.

60	seconds (sec.)	make	1 minute,	m
60	minutes,		1 hour,	h
24	hours,		1 day,	d
365	days,		1 common year,	c. y
366	days,		1 leap year,	l. y

For what is Dry Measure used? Repeat the Table. How cubic inches in a gallon, Dry Measure? What is Time? Repe Table?

Also,

7 days	make	1 week,	w.
12 calendar months (mo.)		1 year,	y.
100 years,		1 century,	C.

99. The calendar months, their names, order, and number of days, are as follows:

January,	1st	mo.	31 days.	July,	7th	mo.	31 days.
February,	2d	"	28 or 29	August,	8th	"	31 "
March,	3d	"	31 days.	September,	9th	"	30 "
April,	4th	"	30 "	October,	10th	"	31 "
May,	5th	"	31 "	November,	11th	"	30 "
June,	6th	"	30 ".	December,	12th	"	31 "

The exact length of an average year being about 365¼ days, the ¼ day in four years makes an additional day. This day, leap year, makes February 29 days.

100. Leap year may be known by being such that its number can be exactly divided by 4. Thus, 1864, 1868, and 1872 are leap years, and have 366 days.

Mental Exercises.

1. How many seconds in 3 minutes? In 4 minutes?
2. How many minutes in two hours? In 3 hours?
3. How many hours in 2 days? In 3 days?
4. How many days in a common year?
5. How many days in a leap year?

Give the names of the calendar months. The length of an average *year. How* may leap year be known?

6. How many minutes in 120 seconds? In 180 seconds?

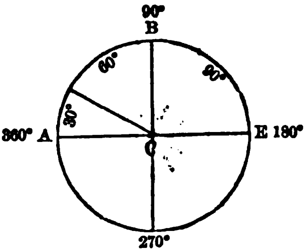
7 How many minutes in 2 hours? In 4 hours?

8. How many hours in 120 minutes? In 240 minutes?

9. How many days in 48 hours? In 72 hours?

10. How many years in 365 days?

Circular Measure.



101. A Circle is a plane figure, bounded by a curved line, all the points of which are equally distant from a point within, called the *center*.

102. The Circumference of a circle is its entire bounding line.

103. An Arc of a circle is any part of the circumference, as AB, or AE.

104. A Diameter of a circle is any straight line drawn through the center, and terminated by the circumference, as AE.

105. Every circle, whatever, is supposed to be divided into 360 equal parts, called *degrees*.

A Quadrant is a fourth of a circle, or an arc of 90 degrees, as AB.

106. The Angle ACB, whose two lines meet at the center C, is measured by the arc of the circle drawn

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What is a Circle? The Circumference of a circle? What is a Arc of a circle? A Diameter? How is every circle divided? Wh is a Quadrant? How is an angle measured?

around that center, included between the opening of those lines. Thus, ACB is an angle of 90 degrees.

A *right angle* is an angle of 90 degrees.

107. Circular Measure is used in measuring angles, and arcs of circles.

Table.

60 seconds (″)	make	1 minute,	′.
60 minutes,		1 degree,	°.
30 degrees,		1 sign,	S.
12 signs, or 360°,		1 circle,	C.

Mental Exercises.

1. How many seconds in 2 minutes? In 3 minutes?
2. How many minutes in 2 degrees? In 3 degrees?
3. How many degrees in 2 signs? In 4 signs?
4. How many signs in 4 circles? In 5 circles?
5. How many minutes in 120 seconds? In 240 seconds?
6. How many degrees in 180 minutes? In 120 minutes?
7. How many signs in 60 degrees? In 90 degrees?
8. How many circles in 360 degrees?

For what is Circular Measure used? Repeat the Table.

Miscellaneous Tables.

108. COUNTING.

12 units	make	1	dozen
12 dozen,		1	gross.
12 gross,		1	great gross.
20 units,		1	score.

109. PAPER.

24 sheets	make	1	quire.
20 quires,		1	ream.
2 reams,		1	bundle.
5 bundles,		1	bale.

110. CAPACITY.

56 pounds of rye	make	1	bushel.
56 pounds of corn,		1	bushel.
60 pounds of wheat,		1	bushel.
60 pounds of beans,		1	bushel.
60 pounds of potatoes,		1	bushel.
60 pounds of clover-seed,		1	bushel.
100 pounds of fish,		1	quintal.
196 pounds of flour,		1	barrel.
200 pounds of beef,		1	barrel.
200 pounds of pork,		1	barrel.

Repeat the Table for Counting? For Paper. For Capacity.

Mental Exercises.

1. How many single things in 2 dozen? In 5 dozen?
2. How many single things in 2 scores? In 6 scores?
3. How many sheets in 2 quires? In 3 quires?
4. How many quires in 4 reams? In 5 reams?
5. How many pounds of wheat in 2 bushels?
6. How many pounds of fish in 5 quintals?
7. How many pounds of pork in 3 barrels?
8. How many bushels in 120 pounds of wheat?
9. How many quintals of fish in 500 pounds?
10. How many barrels of pork in 600 pounds?

REDUCTION.

111. How many inches in 7 yards?

SOLUTION. *Since there are* 3 *feet in* 1 *yard, in* 7 *yards there are* 7 *times* 3 *feet, which are* 21 *feet; and since there are* 12 *inches in* 1 *foot, in* 21 *feet there* 21 *times* 12 *inches, which are* 252 *inches. Therefore, there are in* 7 *yards* 252 *inches.*

We have here *reduced* yards to inches; and the operation, which consists in finding the number of inches equal in value to the given number of yards, is called *Reduction.* Hence,

112. Reduction is the process of changing one number into another of a different denomination, but of equal value.

What is Reduction?

REDUCTION DESCENDING.

113. REDUCTION DESCENDING is the changing of one number into another of a lower denomination of equal value.

Written Exercises.

114. 1. Let it be required to reduce 18 T. 5 cwt. 3 qr. to quarters.

OPERATION.

18 T. 5 cwt. 3 qr.
20
360
5
365, cwt.
4
1460
3
1463, qr.

Since in 1 ton there are 20 hundred, in 18 tons there are 18 times 20 hundred, or 360 hundred, and 5 hundred added make 365 hundred, in 18 T. 5 cwt.

Since in 1 hundred there are 4 quarters, in 365 hundred there are 365 times 4 quarters, or 1460 quarters, and 3 quarters added make 1463 quarters.

Therefore, 18 T. 5 cwt. 3 qr. reduced to quarters is 1463 quarters.

Rule.

Multiply the number of the highest denomination given by the number required of the next lower denomination to make one of that higher, and to the product add the number, if any, of the lower denomination.

Proceed in like manner till the whole is reduced to the required denomination.

What is Reduction Descending? Repeat the Rule. How do you multiply by ½? *Ans.* By taking half of the multiplicand, or by dividing by 2.

Examples.

2. Reduce 27 cwt. 2 qr. 22 lb. to pounds.

3. Reduce 5 fur. 12 rd. 4 yd. to yards.

OPERATION.	OPERATION.
27 cwt. 2 qr. 22 lb.	5 fur. 12 rd. 4 yd.
4	40
108	200
2	12
110, qr.	212, rd.
25	5½
550	1060
220	106
2750	1166
22	4
2772, lb.	1170, yd.

It will be observed, in the second operation, that 5½ times 212 is the same as 5 times 212, and ½ of 212; 5 times 212 is 1060, and ½ of 212 is 106; 1060 and 106 are 1166.

4. Reduce 1 lb. 3 oz. 5 pwt. to pennyweights.
Ans. 305 pwt.
5. Reduce 37 A. 3 R. 12 P. to square yards.
Ans. 183073 sq. yd.
6. In 7 cu. yd. 20 cu. ft. how many cubic inches? *
7. In 4 hogsheads how many quarts?
8. Reduce 45 bu. 3 pk. 5 quarts to quarts.

---

* *For answers*, see corresponding examples in Reduction Ascending.

9. In 10 h. 17 m. 18 s. how many seconds?
10. In 3° 4′ 16″ how many seconds?
11. In 2 T. 9 cwt. how many pounds?
12. Reduce 84 gal. 2 qt. 1 pt. to gills.
13. How many ounces in 12 cwt. 1 qr. 0 lb?
14. How many minutes in 3 wk. 5 d. 10 h.?
15. How many rods in 30 m. 6 fur. 25 rd.?
16. How many square rods in 14 A. 1 R. 35 P.?
17. How many cord feet in 96 cords?
18. How many seconds in 45° 56′ 30″?

115. The preceding rule in its application to United States Money, since the multipliers will be 10, 100, or 1000, admits of an abridged operation. (Art. 58). Thus:

To reduce cents to mills, annex ONE *cipher; to reduce dollars to cents,* TWO *ciphers; and to reduce dollars to mills,* THREE *ciphers.*

To reduce dollars and cents to cents, or dollars, cents, and mills to mills, remove the dollar mark, and the decimal point.

19. Reduce $98 to cents.
20. Reduce 168 cents to mills.
21. Reduce $106 to mills.
22. In $37.19 how many cents?
23. How many cents in $136.05?
24. How many mills in $1325?
25. How many mills in 69 cents?
26. How many mills in $215.09?

How can you reduce dollars to cents? Cents to mills? Dollars to mills? Dollars and cents to cents? Dollars, cents, and mills to mills?

REDUCTION ASCENDING.

116. **Reduction Ascending** is the changing of one number into another of a higher denomination of equal value.

Written Exercises.

117. 1. Let it be required to reduce 1463 quarters to tons.

OPERATION.

4)1463, qr.

20)365, 3 qr.

18 T. 5 cwt.

Ans. 18 T. 5 cwt. 3 qr.

Since in 4 quarters there is 1 hundred, in 1463 quarters there are as many hundred as 4 quarters are contained times in 1463 quarters, which are 365, and 3 quarters remaining.

Since in 20 hundred there is 1 ton, in 365 hundred there are as many tons as 20 hundred are contained times in 365 hundred, which are 18, and five hundred remaining.

Therefore, there are in 1463 quarters, 18 T. 5 cwt. 3 qr.

Rule.

Divide the given number by the number of its denomination required to make one of the next higher, and reserve the remainder, if any.

Proceed in like manner with the quotient, and so continue till the whole is reduced to the required denomination.

The number of the required denomination, with the several remainders, if any, will be the answer.

What is Reduction Ascending? Repeat the Rule.

Examples.

2. Reduce 2772 pounds to hundreds.

OPERATION.

```
25)2772, lb.
   ————
   4)110 . . 22 lb.
    ———
      27 . . 2 qr.
```

Ans. 27 cwt. 2 qr. 22 lb.

3. Reduce 1170 yards to furlongs.

OPERATION.

```
5½)1170, yd.
 2     2
—  ————
11)2340,   hlf. yd.
   ————
 40)212 . . 8 hlf. yd.=4yd.
    ———
      5 . . 12 rd.
```

Ans. 5 fur. 12 rd. 4 yd.

In the second operation, to divide 1170 by 5½, we first reduce both numbers to halves, by multiplying by 2, and have 2340 *halves* to be divided by 11 *halves*, which gives 212, and a remainder of 8 halves = 4.

4. Reduce 305 pennyweights to pounds.
5. Reduce 183073 square yards to acres.
6. Reduce 361152 cubic inches to cubic yards.
7. Reduce 1008 quarts to hogsheads.
8. Reduce 1469 quarts to bushels.
9. Reduce 37038 seconds to hours.
10. Reduce 11056 seconds to degrees.
11. In 4900 pounds, how many tons ?
12. In 2708 gills, how many hogsheads ?
13. In 19600 ounces, how many hundreds ?
14. In 38040 minutes, how many weeks ?
15. In 10265 rods, how many miles ?
16. How many acres in 2315 square rods ?
17. How many cords in 768 cord feet ?
18. How many degrees in 165390 seconds ?

118. The preceding rule in its application to United States Money, since the divisors will be 10, 100, or 1000, admits of an abridged operation (Art. 69.) Thus,

To reduce cents to dollars, point off TWO *figures from the right; to reduce mills to dollars, point off* THREE *figures.*

To reduce mills of two or more places of figures to cents, consider the figure on the right to express mills, and the other cents.

19. Reduce 9800 cents to dollars.
20. Reduce 1680 mills to cents.
21. Reduce 106000 mills to dollars.
22. In 3719 cents, how many dollars?
23. How many dollars in 13605 cents?
24. How many dollars in 1325000 mills?
25. How many cents in 690 mills?
26. How many dollars in 215090 mills?

Miscellaneous Examples.

1. How many pounds in 7 T. 18 cwt.?
2. How many tons in 15800 pounds?
3. How many grains in 2 lb. 11 oz. 20 gr.?
4. How many pounds in 16820 grains?
5. How many quarts in 32 hogsheads?
6. How many hogsheads in 8064 quarts?
7. How many pints in 27 hhds. 18 gal. 2 qts.?
8. In 13756 pints, how many hogsheads?

How can you reduce cents to dollars? Mills to dollars? Mills to *cents?*

9. How many quarts in 17 bu. 1 pk. 4 qt.?
10. How many bushels in 556 quarts?
11. In 12 rd. 5 yd. 2 ft. how many feet?
12. In 215 feet how many rods?
13. Reduce 3 A. 3 R. 20 P. to square yards.
14. Reduce 18755 square yards to acres.
15. Reduce 3 cu. yd. 9 cu. ft. 1480 cu. in. to cu. in.
16. Reduce 157000 cubic inches to cubic yards.
17 Reduce 506 d. 22 h. 40 m. to minutes.
18. Reduce 730000 minutes to days.
19. How many minutes in one common year?
20. How many years in 525600 minutes?
21. Reduce 14° 5′ to seconds.
22. Reduce 50700 seconds to degrees.
23. In 5 reams of paper, how many sheets?
24. In 2400 sheets of paper, how many reams?
25. Reduce $46.05 to mills.
26. In 46050 mills, how many dollars?
27. In 20 eagles, how many cents?
28. In 20000 cents, how many eagles?
29. In $31.99, how many cents?
30. In 3199 cents, how many dollars?
31. In 573 yd. 1 qr. 1 na., how many nails?
32. In 9173 nails, how many yards?
33. In 3 miles, how many inches?
34. In 190080 inches, how many miles?
35. In 57935 mills, how many dollars?
36. In $57.935, how many mills?

37. In a pile of wood 96 feet long, 4 feet high, and 4 feet wide, how many cords? Ans. 12 cords.

How is Reduction Descending performed? *Ans.* By Multiplication. How is Reduction Ascending performed? *Ans.* By Division. How then do their operations differ? How is each kind of Reduction proved? *Ans. Each by the process of the other.*

38. How many square feet in a board 20 feet long and 2 feet wide? Ans. 40 sq. ft.

39. What cost 1 quarter-hundred of beef, at 9 cents a pound?

SOLUTION. *Since* 1 *quarter-hundred is* 25 *pounds,* 1 *quarter-hundred of beef, at* 9 *cents a pound, will cost* 25 *times* 9 *cents, which are* 225 *cents, or* $ 2.25. *Therefore, it will cost* $ 2.25.

40. What cost a barrel of flour, at 6 cents a pound?

41. What is the value of a rectangular lot, 40 rods long and 20 rods wide, at 50 cents a square rod?

42. In a block of stone, 6 feet long and 2 feet square, how many cubic feet? Ans. 24 cu. ft.

43. How many grains in 15 silver spoons, each weighing 6 pwt. 15 gr.? Ans. 2385.

44. What is the cost of leveling a plat of ground, 20 rods long and 20 rods wide, or 20 rods square, at 25 cents a square rod? Ans. $ 100.

45. What is the value of a pile of wood 20 feet long, 6 feet high, and 4 feet wide, at 50 cents a cord foot? Ans. $ 15.

46. What will it cost to pave with marble the floor of a room, 4 yards square, at $ 1.25 per square foot?

47. How many hundred-weight of sugar, at 15 cents a pound, can be bought for $ 20.40?

SOLUTION. *There will be as many pounds as* 15 *cents are contained times in* $ 20.40, *or in* 2040 *cents, which are* 136; *and as many hundred-weight as* 136 *pounds equals, or* 1 *cwt.* 1 *qr.* 11 *lb. Therefore, there can be bought* 1 *cwt.* 1 *qr.* 11 *lb.*

REVIEW QUESTIONS. — Repeat the Table of United States Money? *(71.) Of* Avoirdupois Weight. (83.) Of Troy Weight. (84.) Of *Linear Measure.* (85.)

48. How many hogsheads of molasses, at 12 cents a quart, can be bought for $ 75.12?
Ans. 2 hhd. 30 gal. 2 qt.

49. If the heart beat 70 times in a minute, how many days will be required for it to beat 705100 times? Ans. 7 days.

50. If a ship can sail at the rate of 8 miles an hour, how many weeks will be required for it to sail 4448 miles? Ans. 3 wk. 2 d. 4 h.

51. If a comet move through 10° in one day, how many signs will it move through in 14 days?
Ans. 4 S. 20°.

52. If 9 candles can be made from 1 pound of tallow, how many dozen can be made from 60 pounds?
Ans. 45 dozen.

53. How many minutes in an average year of 365 days, 6 hours? Ans. 525960 m.

54. How many hours from the 12th of January to the 24th of the next April in a common year?
Ans. 2448 h.

SOLUTION. *There will be as many days as are between the dates;* 31 *days in January less* 12 *days* = 19 *days;* 19 *days in Jan.* + 28 *days in Feb.* + 31 *days in Mar.* + 24 *days in Apr.* = 102 *days; and since there are* 24 *hours in* 1 *day, there will be* 102 *times* 24 *hours, or* 2448 *hours. Therefore, there will be* 2448 *hours.*

55. In leap year how many hours from the 8th of January to the 12th of December? Ans. 6792.

REVIEW QUESTIONS. — Repeat the Table of Square Measure. (91.) Of Solid Measure. (95.) Of Time Measure. (98.) Of Circular Measure. (107.)

56. What is the value of 2 R. 10 P. of land, at 6 cents per square foot?

57. What will 3 T. 5 cwt. 20 lb. of iron cost, at 3 cents a pound?

58. At 3 cents a pound, how many tons of iron can be bought for $195.60?

59. How wide must be the floor of a rectangular room, whose length is 8 yards, to require 48 square yards to cover it? Ans. 6 yards.

Here, the 48 is evidently the product of the length and the width, and if we divide the 48 by the length given, or 8, we have the width required.

60. How long must a piece of land be, whose width is 20 rods, to contain just 5 acres? Ans. 40 rods.

61. What cost 645 pounds of potatoes, at 20 cents a peck?

62. How many pounds of potatoes, at 20 cents a peck, can be bought for $8.60?

63. How many days will it take to count a million of dollars, at the rate of a hundred a minute? Ans. 6 d. 22 h. 40 m.

64. How many cords in a pile of wood 9 feet long, 8 feet high, and 4 feet wide? Ans. 2 C. 2 c. f.

65. A pile of wood is 4 feet wide, and 4 feet high; how long must it be to contain 2 cords? Ans. 16 feet.

Here, the product of the length, width, and hight, is equal to the number of cubic feet in 2 cords, or 128 cu. ft. × 2, which is 256 cu. ft.; and if we divide the 256 by the product of the width and hight, which is 16, we have the length required.

Repeat the Table of Liquid Measure. Of Dry Measure.

66. How high must a range of wood, 20 feet in length and 4 feet wide, be piled to make 5 cords?
Ans. 8 feet.

67. How many times will a wheel 18 ft. 4 in. in circumference turn, in going 100 miles?
Ans. 28800 times.

68. How many loaves of bread can be made from 1 barrel and 4 pounds of flour, allowing 10 ounces to a loaf? Ans. 320.

69. How many shingles will be required to cover the side of a roof 30 feet by 20 feet, allowing that 1 thousand just cover 100 square feet?
Ans. 6 thousand.

70. How many days of 10 hours each may be saved for study, in 10 years of 365 days each, by rising 30 minutes earlier each day? Ans. 182 d. 5 h.

COMPOUND NUMBERS.

119. A **Simple Number** is of a single denomination; as, 5 dollars, 8 yards, 4 books.

120. A **Compound Number** is of two or more denominations of the same general nature; as, 5 dollars 50 cents; 4 yards 2 feet 6 inches; 12 hours 20 minutes.

The methods of adding, subtracting, multiplying, and dividing simple numbers apply, with some modifications, to compound numbers.

What is a Simple Number? A Compound Number?

ADDITION.

121. 1. Let it be required to find the sum of 5 cwt. 2 qr. 10 lb., 2 cwt. 2 qr. 17 lb., and 10 cwt. 3 qr. 21 lb.

OPERATION.

	cwt.	qr.	lb.
	5	2	10
	2	2	17
	10	3	21
Ans.	19	0	23

For convenience, we write the numbers so that denominations of the same kind stand in the same column, and begin at the right to add.

21, 17, and 10 lb. are 48 lb., which, reduced, gives 1 qr. 23 lb.; we write the 23 lb and add the 1 qr. in with quarters. 1, 3, 2, and 2 qr. are 8 qr., which reduced gives 2 cwt. 0 qr.; we write the 0, and add the 2 cwt. with the hundred-weights. 2, 10, 2, and 5 cwt. are 19 cwt., which we write. Therefore, the sum required is 19 cwt. 0 qr. 23 lb.

Rule.

Write the numbers so that units of the same denomination shall stand in the same column.

Beginning at the right, add each column, and reduce the sum to the next higher denomination; write the remainder, if any, and add the quotient with the next column.

Proof. The same as in simple numbers.

How do you write the numbers? How do you add? What is the *Rule? The Proof?*

Examples.

(2.)

cwt.	qr.	lb.	oz.
2	1	0	10
5	3	16	15
1	0	1	2
9	0	18	11

(3.)

lb.	oz.	pwt.	gr.
21	8	19	20
10	4	11	16
9	13	0	5
42	2	11	17

(4.)

rd.	yd.	ft.	in.
1	3	2	1
16	4	2	11
20	2	1	10
39	0	0	10

(5.)

M.	fur.	rd.	yd.
6	5	16	3
5	0	23	8
10	3	19	2

(6.)

A.	R.	P.
27	2	31
35	3	24
22	1	17
45	0	29
131	0	21

(7.)

A.	R.	P.
41	2	28
27	1	31
51	0	19
42	1	25

(8.)

C	cu. ft.
5	100
3	27
3	13
7	20
19	32

(9.)

cu. yd.	cu. ft.	cu. in.
6	21	1220
4	10	1531
9	11	160
	8	30

(10.)

T.	cu. ft.
15	25
7	16
	24
3	15
27	0

(11.)

hhd.	gal.	qt.	pt.	gi.
3	16	2	1	1
4	20	3	1	3
	41	1	1	3
6	0	0	1	1
14	16	1	0	0

(12.)

hhd.	gal.	qt.	pt.	gi.
5	0	3	1	2
8	50	2	1	0
6	25	1	0	3
	46	2	1	2

(13.)

bu.	pk.	qt.	pt.
23	3	5	1
10	2	6	1
16	1	3	1
20	1	2	0
70	1	1	1

(14.)

bu.	pk.	qt.	pt.
31	3	6	1
25	1	4	1
32	2	2	1
4	3	0	1

(15.)

d.	h.	m.	s.
5	13	39	42
6	20	29	45
4	22	19	33
4	17	59	59
22	2	28	59

(16.)

d.	h.	m.	s.
4	11	39	28
7	23	19	19
5	21	40	29
2	13	10	32

(17.)

S.	°	′	″
2	10	25	40
1	11	55	0
1	4	3	10
1	0	15	23
5	26	39	13

(18.)

S.	°	′	″
4	14	16	25
1	10	24	0
	12	0	17
3	9	10	8

19. What is the sum of 1 T. 5 cwt. 3 qr. 10 lb., 7 cwt. 20 lb., and 5 T. 3 qr.?

Ans. 6 T. 13 cwt. 3 qr. 5 lb.

20. What is the sum of 4 yd. 2 ft. 6 in., and 5 yd. 1 ft. 11 inches? Ans. 10 yd. 1 ft. 5 in.

21. What is the sum of 9 oz. 18 pwt. 11 gr., 4 oz. 19 pwt. 20 gr., and 1 oz. 23 gr.?

Ans. 1 lb. 3 oz. 19 pwt. 6 gr.

22. Two men leave the same place and travel, the one north 65 m. 5 fur. 30 rd., and the other south 82 m. 6 fur. 16 rd.; how far were they then apart?

Ans. 148 m. 4 fur. 6 rd.

23. A farmer raised in one field 103 bu. 3 pk. 2 qt. of corn; in a second 205 bu. 6 qt., and in a third 310 bu. 3 pk.; how much did he raise in all?

Ans. 619 bu. 3 pk.

24. A ship sailed 15° 50′ 50″ at one time, and 31° 19′ 10″ in the same direction, at another; how many degrees was it then from the place of departure?

Ans. 47° 10′.

SUBTRACTION.

122. 1. Let it be required to find the difference between 19 cwt. 2 qr. 17 lb., and 15 cwt. 3 qr. 15 lb.

OPERATION.

	cwt.	qr.	lb.
	19	2	17
	15	3	15
Ans.	3	3	2

For convenience, we write the subtrahend under the minuend, so that denominations of the same kind stand in the same column, and begin at the right to subtract.

15 lb. from 17 lb. leaves 2 lb.,

How are the numbers written?

which we write. We cannot take 3 qr. from 2 qr., but we can take 1 cwt. from the 19 cwt., leaving 18 cwt., and the 1 cwt. taken is 4 qr., which added to the 2 qr. make 6 qr.; 3 qr. from 6 qr. leaves 3 qr., which we write. 15 cwt. from from 18 cwt. leaves 3 cwt., which we write. Therefore, the difference required is 3 cwt. 3 qr. 2 lb.

Instead of making the upper number, 19 cwt., less by 1, the result would have been the same, if we had increased the corresponding lower number, 15 cwt., by 1. (Art. 48.)

Rule.

Write the less number under the greater, so that denominations of the same kind shall stand in the same column.

Beginning at the right, subtract each denomination of the subtrahend from the corresponding denomination of the minuend.

If the number of any denomination in the subtrahend is greater than that of the same denomination in the minuend, increase the upper number by as many units of that denomination as make one of the next higher, before subtracting; and consider the number of the next higher denomination of the minuend diminished by one.

PROOF. The same as in subtraction of simple numbers.

How do you subtract? How do you proceed when any number of the minuend exceeds the number of the same denomination in the subtrahend? What is the Rule? The Proof?

REVIEW QUESTIONS.—For what is Square Measure used? (91.) Solid Measure? (95.) Liquid Measure? (96.) Dry Measure? (97.) *What is Time?* (98.) For what is Circular Measure used? (107.)

Examples.

(2.)

	T.	cwt.	qr.	lb.
From	14	17	2	16
Take	11	19	3	14
	2	17	4	2

(3.)

cwt	qr.	lb.	oz.
3	0	0	14
1	3	20	15
1	0	4	15

(4.)

	oz.	pwt.	gr.
From	9	16	11
Take	4	19	21
	4	16	14

(5.)

lb.	oz.	pwt.
4	11	6
2	10	11

(6.)

yd.	ft.	in.
4	1	9
3	2	11
	1	10

(7.)

	fur.	rd.	yd.	ft.	
From	7	0	3	2	
Take	4	30	4	1	
	2	9	4½	1	in.
			½	=1	6
	2	9	4	2	6

(8.)

R.	P.	sq. yd.	sq. ft.	sq. in.
3	25	6	8	116
2	24	10	4	100
1	0	26¼	4	16
		¼	=2	36
1	0	26	6	52

(9.)

	hhd.	gal.	qt.	pt.
From	23	50	2	1
Take	12	51	1	2
	10	62	0	1

(10.)

hhd.	gal.	qt.	pt.
19	44	2	0
6	21	3	1

(11.)

	bu.	pk.	qt.
From	37	2	0
Take	19	3	7
	17	2	1

(12.)

bu.	pk.	qt.
54	0	0
16	3	5

(13.)

ch.	bu.
31	19
14	26

(14.)

	c. y.	d.	h.	m.
From	3	300	23	0
Take	1	350	0	40
	1	315	22	20

(15.)

c. y.	d.	h.	m.	s.
16	250	20	45	16
14	300	22	15	17

(16.)

	S.	°	′	″
From	17	6	31	54
Take	14	8	35	56
	2	27	55	58

(17.)

C.	S.	°	′	″
1	10	12	58	43
	11	11	59	40

18. From a chest containing 30 lb. 10 oz. 11 dr. of tea, have been taken 9 lb. 15 oz. 15 dr.; what quantity remains? Ans. 20 lb. 10 oz. 12 dr.

19. From a piece of calico containing 32 yd. 2 qr. 2 na., there have been cut off for a dress 11 yd. 3 qr. 2 na.; how much remains? Ans. 20 yd. 3 qr.

20. From a pile of wood containing 19 C. 86 cu. ft., there have been sold 13 C. 96 c. ft.; how much remains? Ans. 5 C. 118 cu. ft.

21. A farmer raised 106 bushels of wheat, and sold 36 bu. 3 pk. 2 qt. 1 pt.; how much had he left?

Ans. 69 bu. 5 qt. 1 pt.

22. If it were possible for a ship to sail around the world by a direct course, how much of its 360° of circumference would remain to be completed when 196° 55′ 42″ had been accomplished?

Ans. 163° 4′ 18″.

REVIEW QUESTIONS.—What is a Number? (11.) A Simple Number? (119.) A Compound Number? (120.)

123. The rule applies to finding the difference between two dates.

23. A man was born May 16, 1819; how old was he March 9, 1865?

OPERATION.

	y.	mo.	d.
	1865	2	9
	1819	4	16
Ans.	45	9	24

We write the *number* of the year, months, and days of the earlier date under that of the year, months, and days of the later date, and *count as many days to a month as are in the month named in the earlier date.*

Here, the month, that is named in the earlier date, has 31 days.

It is quite common to count every month as containing 30 days, which, in this case, would have given the difference 1 day less, and to that extent wanting in exactness.

24. A note was given April 30, 1865, and paid March 17, 1866; how long did it run?

25. Required the time between the discovery of America by Columbus, Oct. 12, 1492, and the Declaration of Independence, July 4, 1776.

MULTIPLICATION.

124. 1. Let it be required to find the product of 4 bu. 3 pk. 4 qt. 1 pt. multiplied by 7.

OPERATION.

	bu.	pk.	qt.	pt.
	4	3	4	1
				7
Ans.	34	0	7	1

For convenience, we begin with the lowest denomination to multiply. 7 times 1 pt. are 7 pt., which equal 3 qt. 1 pt; we write the 1 pt., and reserve the 3 qt. to add to the product of the quarts. 7 times 4 qt. are 28 qt. which, with the 3 qt. ad-

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Explain the method of finding the difference between two dates.

ded, are 31 qt., or 3 pk. 7 qt.; we write the 7 qt., and reserve the 3 pk. to add to the product of the pecks. 7 times 3 pk. are 21 pk., which, with the 3 pk. added, are 24 pk., or 6 bu. 0 pk.; we write the 0 pk., and reserve the 6 bu. to add to the product of bushels. 7 times 4 bu. are 28 bu., which, with the 6 bu. added, are 34 bu., which we write. Therefore, the product required is 34 bu. 0 pk. 7 qt. 1 pt.

Rule.

Beginning at the right, multiply each denomination of the compound number in its order, and reduce each product to the next higher denomination; write the remainder, if any, and add the quotient to the next product.

Proof. The same as in simple numbers.

Examples.

(2)

cwt.	qr.	lb.	oz.
4	2	16	10
			4
18	2	16	8

(3)

T.	cwt.	qr.	lb.
17	16	3	10
			6

(4)

lb	oz.	pwt.	qr.
	4	10	5
			4

(5)

lb.	oz.	pwt.	qr.
14	9	14	17
			5

With which denomination do you begin to multiply? How do you reduce each product? Repeat the Rule?

(6.)

yd.	ft.	in.
4	1	2
		12
52	2	0

(7.)

m.	fur.	rd.
2	7	30
		6

(8.)

yd.	qr.	na.
5	3	2
		17
99	3	2

9. How many acres in an estate consisting of 5 farms, each measuring 70 A. 2 R. 14 P.?
Ans. 352 A. 3 R. 30 P.

10. How much wood in 4 ranges, each containing 5 cords, 6 cord feet? Ans. 23 C.

11. How much wine in 6 casks, each containing 49 gal. 2 qt. 1 pt.? Ans. 297 gal. 3 qt.

12. How much grain in 15 barrels, each containing 3 bu. 1 pk. 4 qt.?

13. Multiply 64 wk. 17 h. 38 m. by 11.
Ans. 705 wk. 1 d. 1 h. 58 m.

14. Required the weight of 3 dozen of silver spoons, if each dozen weigh 2 lb. 6 oz. 12 pwt. 3 gr.
Ans. 7 lb. 7 oz. 16 pwt. 9 gr.

15. If a planet move 2° 58′ 10″ per day, how far will it move in 56 days?
Ans. 166° 17′ 20″.

16. If you should sleep 8 h. 15 m. 30 s. daily, how much will you sleep in 7 days?

17. What is the weight of 6 chests of tea, each weighing 2 qr. 16 lb. 8 oz.?
Ans. 3 cwt. 3 qr. 24 lb.

18. How much land in 9 equal wood-lots of 113 A. 3 R. 25 P. each? Ans. 1025 A. 0 R. 25 P.

REVIEW QUESTIONS.—How is Addition of Compound Numbers proved? Subtraction of Compound Numbers?

19. If you have for labor and recreation 16 h. 47 m. 30 s. each day, how many full days have you for that purpose in 8 days? Ans. 5 d. 14 h. 20 m.

DIVISION.

125. 1. Let it be required to find the quotient of 34 bu. 0 pk. 7 qt. 1 pt. divided by 7.

OPERATION.

	bu.	pk.	qt.	pt.
7)	34	0	7	1
	4	3	4	1

For convenience, we begin with the highest denomination to divide. One seventh of 34 bu. is 4 bu., with a remainder of 6 bu., equal to 24 pecks; we write the 4 bu., and add the 24 pecks to the 0 pecks in the dividend. 24 pk. and 0 pk. are 24 pk., and one-seventh of 24 pk. is 3 pk., with a remainder of 3 pk., equal to 24 quarts; we write the 3 pk., and add the 24 quarts to the 7 quarts in the dividend. 24 qt. and 7 qt. are 31 qt., and one-seventh of 31 qt. is 4 qt. with a remainder of 3 qt., equal to 6 pints; we write the 4 qt., and add the 6 pints to the 1 pint of the dividend. 6 pt. and 1 pt. are 7 pt., and one-seventh of 7 pt. is 1 pt., which we write.

Rule.

Beginning at the right, divide each denomination of the dividend, in its order; write the quotient, and reduce the remainder, if any, to the next lower denomination, adding the same to that denomination in the dividend, before dividing it.

Proof. The same as in simple numbers.

When the divisor and dividend are both compound, or not of the same denomination, they must be reduced to the same denomination; and then the division is that of simple numbers.

Where do you begin to divide? Repeat the Rule.

Examples.

(2.)

	cwt.	qr.	lb.	oz.
4)	18	2	16	8
	4	2	16	10

(3.)

	T.	cwt.	qr.	lb.
7)	2	15	1	21

(4.)

	lb.	oz.	pwt.	gr.
4)	1	6	0	20
		4	10	5

(5.)

	lb.	oz.	pwt.	gr.
9)	16	3	11	20
	1	9	14	$15\frac{5}{9}$

(6.)

	M.	fur.	rd.
8)	41	6	161
	5	1	32

(7.)

	yd.	ft.	in.
2)	52	2	0

(8.)

	fur.	rd.	yd.
11)	35	10	1
	3	8	2

9. I have 352 A. 3 R. 30 P., which I wish to divide into 5 equal farms; how many acres will there be for each?

10. Divide 69 sq. ft. 36 sq. in. by 9.
Ans. 7 sq. ft. 100 sq. in.

11. Divide 6 T. 11 cwt. 3 qr. 16 lb. by 8.
Ans. 16 cwt. 1 qr. 23 lb.

12. If 9 casks contain 412 gal. 3 qt. 1 pt.; how much is that to a cask? Ans. 45 gal. 3 qt. 1 pt.

13. If 3 dozen spoons weigh 7 lb. 7 oz. 16 pwt. 9 gr., what is the weight of each dozen?
Ans. 2 lb. 6 oz. 12 pwt. 3 gr.

14. If 23 cords of wood be put into 4 equal ranges, how much will be in each range?

126. When the divisor is large, it is often convenient to write out the operation, after the manner of long division.

15. Divide 178 bu. 1 pk. by 23.

OPERATION.

bu. pk.
23) 178 1 (7 bu.
161
17
4
68
1
23) 69 (3 pk.
69

Ans. 7 bu. 3 pk.

16. Divide 286 m. 6 fur. by 31.

OPERATION.

m. fur.
31) 286 6 (9 m.
279
7
8
56
6
31) 62 (2 fur.
62

Ans. 9 m. 2 fur.

17. If a boarding-house requires 175 gal. 2 qt. of molasses in 52 weeks, how much is required a week?
Ans. 3 gal. 1 qt. 1 pt.

18. Divide 675 yd. 3 qr. by 17. Ans. 39 yd. 3 qr.

19. If 15 loads of hay weigh 16 T. 10 cwt., what is the weight of 1 load? Ans. 1 T. 2 cwt.

20. If the moon moves in 29 days 382° 6′ 55″, how much is the daily motion? Ans. 13° 10′ 35″.

When the divisor is large, how is it often convenient to proceed?

Miscellaneous Examples.

1. Reduce 15 cwt. 3 qr. 11 lb. to pounds.

2. Reduce 13120 rods to miles. Ans. 41 miles.

3. Required the cost of 5 tons of iron, at 6 cents a pound. Ans. $600.

4. How many hundred-weight of flour can be bought, at 4 cents a pound, for $7.48? Ans. 1 cwt. 3 qr. 12 lb.

5. If a man travels one day 30 m. 2 fur. 23 rd., and another day 27 m. 3 fur. 17 rd.; how far does he travel in all? Ans. 57 m. 6 fur.

6. If a merchant cut 11 yd. 3 qr. from a piece of calico containing 30 yd. 1 qr. 2 na., how much will remain?

7. If from a cask containing 64 gal. 2 qt. 1 pt. 2 gi. of molasses, there should leak out 11 gal. 3 qt. 1 pt. 1 gi.; how much would remain in the cask? Ans. 52 gal. 3 qt. 0 pt. 1 gi.

8. What is the weight of 3 dozen silver forks, each fork weighing 4 oz. 0 pwt. 6 gr.? Ans. 12 lb. 2 oz. 5 pwt.

9. How much must be paid for 2 ch. 28 bu. of coal, at 25 cents a bushel? Ans. $25.

10. Bought 3 pieces of cloth, each containing 42 yd., of which were sold one piece, and 27 yd. 1 qr. 2 na. of another; what quantity remained? Ans. 56 yd. 2 qr. 2 na.

11. How much older is John than James, John's birth having been on the 20th of December, 1858, and James' on the 10th of August, 1863? Ans. 4 yr. 7 mo. 21 d.

REVIEW QUESTIONS. — What is Reduction? (112.) Reduction *Descending? (113.)* Reduction Ascending? (116.)

12. If a man can lift 201 lb. 12 oz. and that is 8 times what a boy can lift, how much can a boy lift?
Ans. 1 qr. 3 oz. 8 dr.

13. How many square yards of carpeting will it take for a room 26 ft. by 32 ft.?
Ans. 92 sq. yd. 4 sq. ft.

14. If a steamer can cross the Atlantic, the distance being 3174 miles, in 15 days, what is her daily speed? Ans. 211 m. 4 fur. 32 rd.

FACTORING.

127. An **Integer**, or **Integral Number**, is a whole number.

128. A **Prime Number** is a number which cannot be divided without a remainder by any integer different from itself, or 1; as 1, 2, 3, 5, 7.

129. A **Composite Number** is a number which can be divided without a remainder by some other integer different from itself, or 1; as 4, 6, 8, 9.

130. A **Factor** of a number is any integer contained in it without a remainder; thus, 2 is a factor of 4.

131. A **Prime Factor** is a factor which is prime.

132. **Factoring** is the process of finding the factors of a composite number.

What is an Integer? A Prime Number? A Composite Number? A Factor? A Prime Factor? What is Factoring?

133. To find the prime factors of numbers.

1. Find the prime factors of 84.

OPERATION.

2)84
—
2)42
—
3)21
—
7

By trial, we find that 84 is composed of two factors, 2 and 42; of which 2 is prime, and 42 is composite. The 42 we find composed of the factors 2 and 21; of which 2 is prime, and 21 is composite. The 21 is composed of two factors, 3 and 7, both prime.

Therefore, the prime factors required are 2, 2, 3, and 7.

Rule.

Divide the given number by any prime number greater than 1, *that will divide it without a remainder, and the quotient, if composite, in the same manner; and thus continue until the quotient is prime. The divisors and the last quotient will be the prime factors required.*

PROOF. The product of the prime factors will equal the given number, if the work is right.

Examples.

2. What are the prime factors of 44?
Ans. 2, 2, 11.

3. What are the prime factors of 57?

4. What are the prime factors of 68?
Ans. 2, 2, 17.

5. What are the prime factors of 70?
Ans. 2, 5, 7.

How do you divide the given number? If the quotient is composite, how do you proceed? How long do you continue? What will be the prime factors? Repeat the Rule. What is the Proof?

6. What are the prime factors of 95 ?
7. What are the prime factors of 210 ?
Ans. 2, 3, 5, 7.
8. What are the prime factors of 735 ?
Ans. 3, 5, 7, 7.

MULTIPLICATION BY FACTORS.

134. 1. Let it be required to multiply 452 by 63.

OPERATION.

452
7
3164
9
Ans. 28476

63 is equal to 7 times 9; hence 63 times 452 is equal to 9 times 7 times 452. 7 times 452 gives 3164, and 9 times 7 times 452, or 9 times 3164, gives 28476.

Therefore, 452 multiplied by 63 is equal to 28476.

Rule.

Multiply the multiplicand by one factor, and this product by another, and so on, until all the factors have been used; the last product will be the one required.

Examples.

2. Multiply 365 by 35, using its factors.
Ans. 12775.
3. Multiply 205 by 27, or 3 × 9.
4. Multiply 265 by 32, or 4 × 8. Ans. 8480.
5. Multiply 521 by 16, or 4 × 4. Ans. 8336.

When the multiplier is a composite number, how can you multiply? *Ans. By its factors.* What is the Rule?

6. Multiply 1124 by 60, or 10 × 6. Ans. 67440.

Here, we annex a cipher to the multiplicand for the product of it by 10 (Art. 58), and then multiply by the 6; that is, *we annex the cipher to the multiplicand, and multiply by the significant figure.*

7. Multiply 301 by 50, or 10 × 5. Ans. 15050.

8. What cost 42 horses, at $165 each?

9. Multiply 24 d. 8 h. 42 m. 40 s. by 15, using its factors. Ans. 365 d. 10 h. 40 m.

10. Required the weight of 56 bags of grain, each weighing 1 cwt. 0 qr. 12 lb.
Ans. 62 cwt. 2 qr. 22 lb.

11. Required the cost of 249 bales of goods, at $28 per bale.

12. A farm has been divided into 49 lots, each containing 3 A. 2 R. 10 P.; how large was the farm?
Ans. 174 A. 2 R. 10 P.

CANCELLATION.

135. Cancellation is the process of shortening operations in division, by rejecting equal factors from the dividend and divisor.

Cancellation depends upon the principle that

Dividing both the dividend and divisor by the same number will not change the quotient.

To prove this principle, let the dividend be 16 and the divisor 4. Then, 16 ÷ 4 = 4. Now, if we divide the dividend and divisor by some number, as 2, the dividend becomes 8, and the divisor, 2; and 8 ÷ 2 = 4, the same result as before.

What is Cancellation? Upon what principle does Cancellation depend? Prove this principle.

136. To shorten operations by Cancellation.

1. Divide 60 by 15.

OPERATION.

$$\frac{60}{15} = \frac{\cancel{3} \times 20}{\cancel{3} \times 5} = 4 \text{ Ans.}$$

For convenience, we indicate the division, by writing the dividend over the divisor, (Art. 66.) Factoring, the dividend becomes 3 × 20, and the divisor 3 × 5. Canceling, by crossing the factor 3, common to both dividend and divisor, we have $\frac{20}{5}$, or 20 ÷ 5, equal to 4. Therefore, the quotient required is 4.

2. Divide 2 times 45 by 9.

OPERATION.

$$\frac{45 \times 2}{9} = \frac{\cancel{9} \times 5 \times 2}{\cancel{9}} = 10 \text{ Ans.}$$

Indicating the division, and factoring, the dividend becomes 9 × 5 × 2. Canceling the factor 9, common to both dividend and divisor, we have left in the dividend 5 × 2, or 10, which is the quotient.

3. Divide 16 times 21 by 8 times 14.

OPERATION.

$$\frac{\overset{\cancel{2}}{\cancel{16}} \times \overset{3}{\cancel{21}}}{\cancel{8} \times \underset{\cancel{7}}{\cancel{14}}} = 3 \text{ Ans.}$$

Canceling the factor 8, common to both divisor and dividend, we have left in the dividend 2, in place of 16; canceling the factor 2, common to both divisor and dividend, we have left in the divisor 7 in place of 14; and canceling the factor 7, common to the dividend and divisor, we have left in the *dividend 3*, which is the quotient.

Rule.

Cancel all the factors common to both dividend and divisor, and then divide the product of the remaining factors of the dividend by the product of those remaining in the divisor.

When a factor is canceled, 1 is understood to remain, but does not require to be written, except in the quotient, when there are no other factors.

Examples.

4. Divide the product of 16 × 14 by 8 × 7.
Ans. 4.

5. Divide the product of 21 × 15 × 5 by 18 × 10 × 3.
Ans. $2\frac{11}{21}$.

6. Divide the product of 33 × 35 × 6 by 11 × 7 × 3.

7. Divide the product of 72 × 8 by 12 × 6.
Ans. 8.

8. How many bushels of potatoes, at 60 cents a bushel, must be given in exchange for 4 calico dresses, each containing 12 yards, at 30 cents a yard?

9. How many bushels of peaches, at 40 cents a peck, must be given for 5 bushels of apples at 32 cents a peck?
Ans. 4 bushels.

10. Exchanged 3 firkins of butter, each containing 56 pounds, at 24 cents a pound, for 6 bags of coffee, each containing 32 pounds. How much was the coffee a pound?
Ans. 21 cents.

What is the Rule? When a factor is canceled, what remains?

ANALYSIS.

137. Analysis, in Arithmetic, is the process of solving a question or example, by considering the relation of its numbers.

Mental Exercises.

1. If 6 oranges cost 18 cents, what will 8 oranges cost?

SOLUTION. *If* 6 *oranges cost* 18 *cents,* 1 *orange will cost* 1 *sixth of* 18 *cents, which is* 3 *cents, and* 8 *oranges will cost* 8 *times* 3 *cents, which are* 24 *cents. Therefore, if* 6 *oranges cost* 18 *cents,* 8 *oranges will cost* 24 *cents.*

2. If 5 peaches cost 10 cents, how much will 10 peaches cost?
3. If 7 yards of velvet cost $28, how much will 9 yards cost?
4. If 8 pounds of sugar cost 72 cents, how much will 10 pounds cost?
5. When 36 cents are paid for 4 quarts of berries, how much must be paid for 7 quarts?
6. What will 10 quarts of milk cost, if 5 quarts cost 40 cents?
7. When $6 will buy 24 yards of cloth, how many yards will $8 buy?
8. If 9 pounds of veal cost 54 cents, how many pounds can be bought for 42 cents?

What is Analysis?

9. If 7 plows can be bought for $77, how many can be bought for $44?

10. When $8 will buy 40 rakes, how many can be bought for $11?

11. If 12 men can do a piece of work in 6 days, how many days will it take 9 men to do the same?

SOLUTION. *If* 12 *men can do the work in* 6 *days, it will take* 1 *man* 12 *times* 6 *days, or* 72 *days, to do it, and* 9 *men* 1 *ninth of* 72 *days, or* 8 *days.*

12. If 7 men can dig a ditch in 8 days, how many days will be required for 14 men to dig it?

13. If 6 horses eat a bag of oats in 10 days, how many horses will eat the same quantity in 15 days?

14. How long will it take 5 men to reap a field, if it take 15 men 3 days to reap it?

15. How long will it take 9 men to build a wall, if it take 6 men 6 days to build it?

16. If 7 books cost $21, what will 11 books cost?

17. When 8 barrels of flour will pay for 12 yards of broadcloth, at $6 a yard, how much is the flour per barrel?

18. If 5 men can hoe a field of corn in 9 days, in what time can 15 men hoe it?

19. When 10 oranges will pay for 20 apples, at 2 cents each, what is the price of each orange?

20. When 10 bushels of wheat will pay for 5 cords of wood, at 4 dollars per cord, how much is the wheat per bushel?

REVIEW QUESTIONS.—What is an Integer? (127.) A Prime Number? (128.) A Composite Number? (129.) A Factor? (130.)

Written Exercises.

1. If 9 acres of land produce 315 bushels of wheat, how many bushels will 7 acres produce?

OPERATION.

$$\frac{\overset{35}{\cancel{315}} \times 7}{\cancel{9}} = 245 \text{ bu. Ans.}$$

If 9 acres of land produce 315 bushels of wheat, 1 acre will produce 1 ninth of 315 bushels, or 35 bushels, and 7 acres will produce 7 times 35 bushels, or 245 bushels.

2. If 10 yards of cloth cost $15, what will 8 yards cost? Ans. $12.

3. If 5 pounds of cheese cost 45 cents, what will 18 pounds cost? Ans. $1.62.

4. If 26 yards of cloth cost $78, what will 17 yards cost?

5. If 72 bushels of apples cost $36, what will 84 bushels cost? Ans. $42.

6. If 8 bushels of wheat cost 12 dollars, what will 20 bushels cost? Ans. $30.

7. If a man can earn in 5 months $176.25, how much can he earn in 12 months, or 1 year? Ans. $423.

8. When 3 books cost $1.14, what will 3 dozen cost? Ans. $13.68.

9. If 3 dozen books cost $13.68, what will 3 books cost?

10. When $12.75 will purchase 17 yards of cloth, how many yards will $2.25 purchase?

Review Questions.—What is a Prime Factor? (131.) Factoring? (132.) What is Cancellation? (135.) Upon what principle *does it depend*? (135.)

11. How far can a man travel in 7 days, at the rate of 244 miles in 4 days? Ans. 427 miles.

12. If 14 horses eat a load of hay in 14 days, how many horses would be required to eat the hay in 7 weeks? Ans. 4 horses.

13. Bought 72 yards of broadcloth for 160 bushels of wheat, at $1.80 per bushel; how much was the broadcloth a yard? Ans. $4.

14. If I give $1887 for 148 barrels of flour, how many barrels can I buy for $63.75. Ans. 5 barrels.

15. If 7 men can build a house in 84 days, how many men could build it in 21 days?
Ans. 28 men.

16. When $30 is paid for 25 pounds of tea, how much must be paid for 40 pounds?

17. A drover exchanged 25 horses for 220 sheep, at $5.50 each; how much did he get for each of his horses? Ans. $121.

18. If 5 tons of hay cost $106.25, what will 13 tons cost? Ans. $276.25.

19. In what time will 48 men do a piece of work that 12 men can do in 24 days? Ans. 6 days.

20. When 15 pounds of sugar can be bought for $1.80, what will 18 pounds cost?

21. How far can a man travel in 18 days, at the rate of 378 miles in 6 days? Ans. 1134 miles.

22. If a man travel 1134 miles in 18 days, how many miles will he travel in 6 days?

23. How many pounds of coffee can be bought for $21, if 30 pounds cost $6? Ans. 105 pounds.

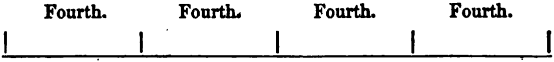
REVIEW QUESTIONS.—What is Analysis in Arithmetic? (137.) What is Arithmetic? (15.) Addition? (39.) Subtraction? (43.) Multiplication? (50.) Division? (60.) Reduction? (112.)

COMMON FRACTIONS.

138. If a unit, as 1 orange, or 1 apple, be divided into two equal parts, each of these parts is *one half*.

If a unit be divided into three equal parts, one of these parts is *one third;* two of them, *two thirds;* etc.

If a unit be divided into four equal parts, one of these parts is *one fourth;* two of them, *two fourths;* three of them, *three fourths;* etc.



In like manner, if a unit be divided into five equal parts, the parts are *fifths;* if into six equal parts, *sixths;* and so on. Such parts of a unit are called *fractions*. Hence,

139. A **Fraction** is one or more equal parts of a unit; as one half, two thirds.

140. The DENOMINATOR of a fraction is the number which shows into how many equal parts the unit is divided. Thus, three is the denominator of two thirds.

If a unit be divided into two equal parts, what is one of the equal parts called? Into three equal parts? Into four? Into five? What is a Fraction? The Denominator?

141. The Numerator of a fraction is the number which shows how many of the equal parts of the unit are taken. Thus, two is the numerator of two thirds.

142. A **Common Fraction** is a fraction expressed by writing the numerator above, and the denominator below, a dividing line; as,

One half, written	$\frac{1}{2}$	Two fifths, written	$\frac{2}{5}$
One third, "	$\frac{1}{3}$	Three fifths, "	$\frac{3}{5}$
Two thirds, "	$\frac{2}{3}$	Four fifths, "	$\frac{4}{5}$
One fourth, "	$\frac{1}{4}$	One seventh, "	$\frac{1}{7}$
Two fourths, "	$\frac{2}{4}$	Three eighths, "	$\frac{3}{8}$
Three fourths written,	$\frac{3}{4}$	Five ninths, "	$\frac{5}{9}$
One fifth, "	$\frac{1}{5}$	Seven tenths, "	$\frac{7}{10}$

143. The Terms of a fraction are the numerator and denominator.

The Unit of a fraction is the unit, or single thing divided into equal parts.

144. A whole number may be expressed in a fractional form, by writing 1 under it, for a denominator. Thus,

2 may be written $\frac{2}{1}$, and read, 2 ones.
3 may be written $\frac{3}{1}$, and read, 3 ones.
7 may be written $\frac{7}{1}$, and read, 7 ones.
10 may be written $\frac{10}{1}$, and read, 10 ones.

What is the Numerator of a fraction? What is a Common Fraction? What are the Terms of a Fraction? How may a whole number be expressed in a fractional form?

EXERCISES.

Express by figures:—

1. Two sixths. Ans. $\frac{2}{6}$
2. Three sevenths.
3. Five eighths.
4. Six ninths.
5. Two tenths.
6. Seven elevenths.
7. Eight twelfths.
8. Nine sixteenths.
9. Seven ones.
10. Four ninths.
11. Eight seventeenths.
12. Fifteen twentieths.
13. Ten thirtieths.
14. Six ninths.
15. Eleven ones.
16. Seventeen fortieths.

Write and read the following:—

17. $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{12}$, $\frac{8}{15}$.
18. $\frac{10}{17}$, $\frac{16}{41}$, $\frac{13}{21}$, $\frac{12}{13}$
19. $\frac{17}{15}$, $\frac{11}{13}$, $\frac{2}{4}$, $\frac{3}{8}$.
20. $\frac{32}{16}$, $\frac{17}{1}$, $\frac{14}{5}$, $\frac{6}{75}$.
21. $\frac{31}{1}$, $\frac{60}{71}$, $\frac{3}{25}$, $\frac{5}{11}$.
22. $\frac{6}{19}$, $\frac{7}{21}$, $\frac{11}{51}$, $\frac{14}{1}$.

145. A PROPER FRACTION is one whose numerator is less than the denominator; as $\frac{3}{7}$, $\frac{7}{8}$.

146. An IMPROPER FRACTION is one whose numerator equals or exceeds the denominator; as $\frac{5}{5}$, $\frac{8}{3}$.

147. A MIXED NUMBER is a whole number with a fraction; as $3\frac{1}{4}$; read, three and one fourth.

148. A fraction indicates division (Art. 66), the numerator answering to the dividend, and the denominator to the divisor. Hence,

The VALUE *of a fraction is the quotient of the numerator divided by the denominator.*

What is a Proper Fraction? An Improper Fraction? A Mixed Number? What does a fraction indicate? What is the value of a fraction?

Thus, the quotient of 1 divided by 2 is $\frac{1}{2}$; 3 divided by 4 is $\frac{3}{4}$; and $\frac{4}{2}$ is 2.

It has been shown, (Art. 135), that canceling equal factors in the dividend and divisor does not change the quotient. Hence,

Dividing both terms of a fraction by the same number does not change its value.

And it also follows, that

Multiplying both terms of a fraction by the same number does not change its value.

$$\text{Thus, } \frac{4}{2}, \text{ or } \frac{4 \div 2}{2 \div 2} = 2, \text{ and } \frac{4}{2}, \text{ or } \frac{4 \times 2}{2 \times 2} = 2.$$

Mental Exercises.

1. How much is $\frac{1}{2}$ of 6? Of 8? Of 12?
2. How much is $\frac{1}{3}$ of 9? Of 12? Of 18?
3. How much is $\frac{1}{4}$ of 8? Of 12? Of 20?
4. How much is $\frac{1}{5}$ of 5? Of 15? Of 25?
5. How much is $\frac{1}{7}$ of 7? Of 14? Of 35?
6. How much is $\frac{1}{6}$ of 12? Of 24? Of 48?
7. How much is $\frac{1}{10}$ of 20? Of 70? Of 100?
8. How much is $\frac{1}{12}$ of 24? Of 36? Of 72?
9. How much is $\frac{1}{6}$ of 30? $\frac{1}{8}$ of 64? $\frac{1}{11}$ of 77?
10. How much is $\frac{1}{9}$ of 27? $\frac{1}{15}$ of 30? $\frac{1}{20}$ of 60?

What effect has dividing both terms of a fraction by the same number? What also follows?

11. How much is $\frac{2}{3}$ of 30 ?

Solution. *Since* 1 *third of* 30 *is* 10, 2 *thirds of* 30 *is* 2 *times* 10, *or* 20. *Therefore,* $\frac{2}{3}$ *of* 30 *is* 20.

12. How much in $\frac{3}{5}$ of 15 ? Of 25 ? Of 45 ?
13. How much is $\frac{2}{9}$ of 36 ? Of 45 ? Of 27 ?
14. How much is $\frac{3}{4}$ of 40 ? Of 12 ? Of 28 ?
15. How much is $\frac{3}{7}$ of 21 ? Of 49 ? Of 42 ?
16. How much is $\frac{5}{6}$ of 18 ? Of 30 ? Of 48 ?
17. How much is $\frac{3}{8}$ of 24 ? Of 32 ? Of 64 ?
18. How much is $\frac{4}{5}$ of 20 ? Of 40 ? Of 50 ?
19. How much is $\frac{5}{8}$ of 32 ? Of 40 ? Of 56 ?
20. If a pound of sugar cost 15 cents, how much will 1 third of a pound cost ? 2 thirds ?
21. Paid $ 60 for a cow, and 2 twelfths as much for some hay ; how much did the hay cost ?
22. What part of 7 is 2 ?

Solution. *Since* 1 *is* 1 *seventh of* 7, 2 *is* 2 *times* 1 *seventh of* 7, *or* 2 *sevenths of* 7. *Therefore,* 2 *is* $\frac{2}{7}$ *of* 7.

23. What part of 5 is 3 ? 4 ? 2 ?
24. What part of 8 is 3 ? 5 ? 7 ?
25. What part of 9 is 2 ? 7 ? 8 ?
26. What part of 10 is 9 ? 3 ? 7 ?
27. What part of 16 yards is 5 yards ? 11 yards ?
28. What part of $ 20 is $ 7 ? $ 13 ? $ 17 ?
29. What part of 10 pounds is 5 pounds ? 8 pounds ?
30. What part of 27 tons is 3 tons ? 9 tons ?
31. What part of $32 is $12 ? $4 ? $16 ?
32. What part of 40 bushels is 20 bushels ?
33. What part of 36 hogsheads is 9 hogsheads ?

Explain the solution of example 11. Of example 22.

REDUCTION.

149. Reduction of Fractions is the process of changing their form, without changing their value.

Case I.

150. To reduce a fraction to its lowest terms.

A fraction is in its lowest or smallest terms, when the numerator and denominator have no common factor greater than 1.

1. Let it be required to reduce $\frac{4}{10}$ to its lowest terms.

OPERATION.

$$\frac{4}{10}=\frac{2\times\cancel{2}}{5\times\cancel{2}}=\frac{2}{5},\ \text{Ans.}$$

Since the numerator and denominator of a fraction answer to the dividend and divisor in division (Art. 148), dividing both terms by the same number, or canceling equal factors in both (Art. 135), will not change the value expressed. Now the only factor greater than 1, common to both terms of the given fraction, is 2, and canceling, we have $\frac{2}{5}$. Therefore, $\frac{4}{10}$ in its lowest terms is $\frac{2}{5}$.

Rule.

Cancel in the numerator and denominator all the factors common to both.

What is Reduction of Fractions? When is a fraction in its lowest terms? Why is not the value of a fraction changed by dividing both terms by the same number? What is the Rule?

Examples.

Reduce to their lowest terms: —

2. $\frac{9}{12}$. Ans. $\frac{3}{4}$.
3. $\frac{77}{84}$. Ans. $\frac{11}{12}$.
4. $\frac{49}{343}$. Ans. $\frac{1}{7}$.
5. $\frac{132}{264}$.
6. $\frac{147}{168}$.
7. $\frac{182}{196}$. Ans. $\frac{13}{14}$.
8. $\frac{60}{125}$. Ans. $\frac{12}{25}$.
9. $\frac{72}{94}$. Ans. $\frac{36}{47}$.
10. $\frac{252}{308}$.
11. $\frac{840}{1120}$.

Case II.

151. To reduce a whole number to a fraction with a given denominator.

1. Let it be required to reduce 8 to a fraction whose denominator is 7.

OPERATION.

$$\frac{8 \times 7}{7} = \frac{56}{7}, \text{ Ans.}$$

Since in 1 there are 7 sevenths, in 8 there are 8 times 7 sevenths, or $\frac{56}{7}$. Therefore, 8 is equal to $\frac{56}{7}$.

Rule.

Multiply the whole number by the given denominator, and write the product over the denominator.

Examples.

Reduce

2. 14 to fourths. Ans. $\frac{56}{4}$.
3. 19 to fifths. Ans. $\frac{95}{5}$.
4. 31 to eighths.
5. 98 to thirds. Ans. $\frac{294}{3}$.
6. 70 to thirty-fourths. Ans. $\frac{2380}{34}$.
7. 135 to seventeenths.

Explain the operation of reducing a whole number to a fraction with a given denominator.

8. 9 is equal to what fraction having 15 for a denominator? Ans. $\frac{135}{15}$.

9. Reduce 131 to a fraction whose denominator shall be 11. Ans. $\frac{1441}{11}$.

Case III.

152. To reduce a mixed number to an improper fraction.

1. Let it be required to reduce $5\frac{2}{3}$ to thirds.

OPERATION.

$$5\frac{2}{3} = \frac{5 \times 3}{3} + \frac{2}{3} = \frac{15 + 2}{3} = \frac{17}{3}, \text{ Ans.}$$

Since in 1 there are 3 thirds, in 5 there are 5 times 3 thirds; and 5 times 3 thirds plus $\frac{2}{3}$, or 15 thirds plus 2 thirds, are 17 thirds, or $\frac{17}{3}$. Therefore, $5\frac{2}{3}$ is equal to $\frac{17}{3}$.

Rule.

Multiply the integral part of the mixed number by the denominator of the fractional part; to the product add the numerator; and write the result over the denominator.

Examples.

Reduce to an improper fraction:—

2. $7\frac{4}{9}$.	Ans. $\frac{67}{9}$.	6.	$12\frac{15}{17}$.	Ans. $\frac{219}{17}$.
3. $16\frac{3}{7}$.	Ans. $\frac{115}{7}$.	7.	$19\frac{12}{18}$.	Ans. $\frac{354}{18}$.
4. $9\frac{2}{11}$.	Ans. $\frac{101}{11}$.	8.	$14\frac{3}{10}$.	Ans. $\frac{143}{10}$.
5. $6\frac{4}{13}$.		9.	$161\frac{21}{40}$.	

Explain the operation of reducing a mixed number to an improper fraction. Repeat the Rule.

10. In $10\frac{1}{4}$ dollars, how many fourths of a dollar? Ans. $\frac{41}{4}$.

11. In 221 bushels, how many elevenths of a bushel? Ans. $\frac{2431}{11}$.

Case IV.

153. To reduce an improper faction to a whole or mixed number.

1. Let it be required to reduce $\frac{17}{3}$ to a whole or mixed number.

OPERATION.

$\frac{17}{3} = 5\frac{2}{3}$, Ans.

Since there are 3 thirds in 1 unit, there will be in 17 thirds as many units as 3 is contained times in 17, or $5\frac{2}{3}$. Therefore, $\frac{17}{3}$ is equal to $5\frac{2}{3}$.

Rule.

Divide the numerator by the denominator.

Examples.

Reduce to a whole or mixed number:—

2. $\frac{67}{9}$. Ans. $7\frac{4}{9}$.
3. $\frac{101}{7}$. Ans. $14\frac{3}{7}$.
4. $\frac{343}{13}$.
5. $\frac{405}{9}$
6. $\frac{108}{4}$. Ans. 27.
7. $\frac{405}{9}$. Ans. 45.
8. $\frac{106}{106}$.
9. $\frac{245}{8}$

10. In $\frac{100}{3}$ of a dollar, how many dollars? Ans. $\$33\frac{1}{3}$.

11. What is the value of $\frac{8224}{16}$ miles? Ans. 514 miles.

Explain the operation of reducing an improper fraction to a whole *or mixed* number. Repeat the Rule.

COMMON DENOMINATOR.

154. Fractions have a *common denominator* when they have the same number for a denominator.

1. Let it be required to reduce $\frac{2}{5}$ and $\frac{3}{7}$ to fractions having a common denominator.

OPERATION.

$$\left.\begin{array}{l}\frac{2}{5}=\frac{2\times7}{5\times7}=\frac{14}{35}\\[4pt] \frac{3}{7}=\frac{3\times5}{7\times5}=\frac{15}{35}\end{array}\right\}\text{Ans.}$$

Since multiplying both the numerator and denominator of a fraction by the same number does not change its value (Art. 148), we multiply both terms of the fraction $\frac{2}{5}$ by 7, the denominator of the second fraction, and have $\frac{2}{5}=\frac{14}{35}$; and both terms of the fraction $\frac{3}{7}$ by 5, the denominator of the first fraction, and have $\frac{3}{7}=\frac{15}{35}$. Therefore, $\frac{2}{5}$ and $\frac{3}{7}$ are equal to $\frac{14}{35}$ and $\frac{15}{35}$.

2. Let it be required to reduce $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{3}{12}$ to fractions having a common denominator.

OPERATION.

$$\left.\begin{array}{r}\frac{2}{3}=\frac{2\times4}{3\times4}=\frac{8}{12}\\[4pt] \frac{5}{6}=\frac{5\times2}{6\times2}=\frac{10}{12}\\[4pt] \frac{3}{12}\end{array}\right\}\text{Ans.}$$

Here, by multiplying both terms of $\frac{2}{3}$ by 4, and of $\frac{5}{6}$ by 2, we have $\frac{8}{12}$ and $\frac{10}{12}$, which have the same number for a denominator as the $\frac{3}{12}$. Therefore, the required fractions are $\frac{8}{12}$, $\frac{10}{12}$, and $\frac{3}{12}$.

Rule.

Multiply both terms of each fraction by all the denominators except its own. Or,

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When have fractions a Common Denominator? Explain the manner of finding the Common Denominator by the first operation. By the second operation. Repeat the Rule.

Multiply both terms of one or more of the fractions by such a number as will make all the denominators alike.

If there are whole or mixed numbers with the given fractions, they must first be reduced to improper fractions.

Examples.

Reduce to equivalent fractions having a common denominator: —

3. $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$. Ans. $\frac{40}{60}$, $\frac{45}{60}$, $\frac{48}{60}$.
4. $\frac{1}{8}$, $\frac{2}{3}$, and $\frac{1}{24}$. Ans. $\frac{3}{24}$, $\frac{16}{24}$, $\frac{1}{24}$.
5. $\frac{2}{5}$, 4, and $\frac{3}{10}$. Ans. $\frac{4}{10}$, $\frac{40}{10}$, $\frac{3}{10}$.
6. $\frac{3}{4}$, $\frac{5}{9}$, and $\frac{7}{12}$. Ans. $\frac{27}{36}$, $\frac{20}{36}$, $\frac{21}{36}$.
7. $\frac{1}{3}$, $\frac{3}{5}$, and $\frac{4}{15}$.
8. $\frac{3}{8}$, $\frac{2}{16}$, and $\frac{1}{4}$.
9. $\frac{5}{6}$ and $\frac{3}{8}$.
10. $\frac{3}{4}$, $\frac{7}{8}$, and $\frac{3}{2}$.
11. $2\frac{1}{2}$, 3, $\frac{5}{4}$, and $\frac{3}{8}$. Ans. $\frac{20}{8}$, $\frac{24}{8}$, $\frac{10}{8}$ $\frac{3}{8}$.

ADDITION.

155. Fractions may be added by means of a common denominator.

Thus, we can add *halves* and *halves, thirds* and *thirds, fourths* and *fourths,* etc.; but we can not add directly *halves* and *thirds, thirds* and *fourths,* etc., any more than we can add other things of different kinds, as dollars and days.

By what means may fractions be added?

1. Let it be required to find the sum of $\frac{1}{5}$, $\frac{3}{5}$, and $\frac{4}{5}$.

OPERATION.

$$\frac{1}{5}+\frac{3}{5}+\frac{4}{5}=\frac{1+3+4}{5}=\frac{8}{5}=1\frac{3}{5}\text{ Ans.}$$

The sum of 1 *fifth*, 3 *fifths*, and 4 *fifths* is 8 *fifths*, or $\frac{8}{5}$, which reduced gives $1\frac{3}{5}$.

2. Let it be required to find the sum of $\frac{3}{4}$ and $\frac{4}{5}$.

OPERATION.

$$\frac{3}{4}+\frac{4}{5}=\frac{15}{20}+\frac{16}{20}=\frac{15+16}{20}=\frac{31}{20}=1\frac{11}{20}.\text{ Ans.}$$

Since numbers must be of the same name or kind, in order to be added (Art. 39), we reduce the given fractions to equivalent fractions having a common denominator, and have 15 *twentieths* and 16 *twentieths*, which added gives 31 *twentieths*, $\frac{31}{20}$, or, by reduction, $1\frac{11}{20}$. Therefore, the sum of $\frac{3}{4}$ and $\frac{4}{5}$ is $1\frac{11}{20}$.

Rule.

Reduce the fractions, if necessary, to equivalent fractions having a common denominator, and write the sum of the numerators over the common denominator.

Examples.

Find the sum of

3. $\frac{4}{9}$, $\frac{7}{9}$, and $\frac{11}{9}$. Ans. $2\frac{4}{9}$.
4. $\frac{8}{15}$ and $\frac{8}{20}$. Ans. $\frac{56}{60}$.
5. $\frac{3}{5}$ and $\frac{4}{7}$. Ans. $1\frac{6}{35}$.
6. $\frac{4}{7}$, $\frac{2}{7}$, $\frac{5}{7}$, and $\frac{6}{7}$.
7. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{7}{8}$, and $\frac{1}{4}$.
8. $\frac{1}{8}$, $\frac{3}{10}$, and $\frac{4}{12}$. Ans. $\frac{91}{120}$.
9. $\frac{7}{10}$, $\frac{11}{12}$, and $\frac{4}{9}$. Ans. $2\frac{11}{180}$.
10. $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{7}$. Ans. $2\frac{11}{84}$.
11. $\frac{1}{8}$, $\frac{2}{3}$, and $\frac{3}{4}$.
12. $\frac{4}{5}$, $\frac{1}{3}$, $\frac{3}{8}$, and $\frac{7}{10}$.

What is the Rule?

156. When there are mixed numbers, *the fractions and whole numbers may be added separately; and, then, their sums may be added.*

13. What is the sum of $3\frac{1}{4}$, $6\frac{1}{2}$, and $2\frac{3}{8}$?

OPERATION.

$$\tfrac{1}{4} + \tfrac{1}{2} + \tfrac{3}{8} = \tfrac{2}{8} + \tfrac{4}{8} + \tfrac{3}{8} = \tfrac{9}{8} = 1\tfrac{1}{8};$$
$$3 + 6 + 2 = 11;\ 11 + 1\tfrac{1}{8} = 12\tfrac{1}{8},\ \text{Ans.}$$

14. What is the sum of $5\frac{2}{3}$, $6\frac{7}{8}$, and $4\frac{1}{2}$?
Ans. $17\frac{1}{24}$.

15. What is the sum of $17\frac{1}{2}$, $2\frac{5}{8}$, and 3?
Ans. $23\frac{1}{8}$.

16. Required the sum of $5\frac{4}{9}$, 6, and $7\frac{3}{7}$.
Ans. $18\frac{55}{63}$.

17. Bought a handkerchief for $\frac{5}{8}$ of a dollar, a vest for $2\frac{1}{2}$ dollars, a pair of gloves for $\frac{5}{16}$ of a dollar, and a hat for $4\frac{1}{4}$ dollars; how much did the whole cost? Ans. $\$7\frac{11}{16}$.

18. A merchant has sold from a piece of cloth, 12, $4\frac{2}{3}$, $8\frac{1}{7}$, and $2\frac{5}{7}$ yards; how many yards is this in all? Ans. $27\frac{11}{21}$.

19. What is the sum of $\$9\frac{1}{5}$, $\$12\frac{1}{3}$, $\$10\frac{2}{3}$, and $\$7\frac{3}{4}$?

20. I have bought three lots of coal, weighing respectively, $1\frac{3}{5}$, $\frac{4}{15}$, and $9\frac{3}{20}$ tons; how much is there in all? Ans. $11\frac{1}{60}$ tons.

21. A farmer sold sheep for $\$62\frac{7}{8}$, cattle for $\$102\frac{3}{5}$, and a horse for $\$125\frac{3}{4}$; how much did he receive for all? Ans. $\$291\frac{9}{40}$.

How may the addition be performed when there are mixed numbers? How may a whole number be expressed in a fractional form? (Art. *144.)*

SUBTRACTION.

157. One fraction may be subtracted from another, by means of a common denominator.

Thus, we can subtract *halves* from *halves*, *thirds* from *thirds*, *fourths* from *fourths*, etc.; but we can not subtract directly *thirds* from *halves*, *fourths* from *thirds*, etc., any more than we can subtract days from dollars, which are of different names or kinds.

1. Let it be required to find the difference between $\frac{17}{18}$ and $\frac{14}{18}$.

OPERATION.

$$\tfrac{17}{18} - \tfrac{14}{18} = \tfrac{17-14}{18} = \tfrac{3}{18} = \tfrac{1}{6}, \text{ Ans.}$$

The difference between 17 *eighteenths and* 14 *eighteenths* is 3 *eighteenths* or $\frac{3}{18}$, which reduced gives $\frac{1}{6}$. Therefore, the difference required is $\frac{1}{6}$.

2. Let it be required to find the difference between $\frac{7}{8}$ and $\frac{3}{4}$.

OPERATION.

$$\tfrac{7}{8} - \tfrac{3}{4} = \tfrac{7}{8} - \tfrac{6}{8} = \tfrac{7-6}{8} = \tfrac{1}{8}, \text{ Ans.}$$

Since, in order to subtract one number from another, the two numbers must be of the same name or kind (Art. 43), we reduce the given fractions to equivalent fractions having a common denominator, and have 7 *eighths* and 6 *eighths*, whose difference is 1 *eight*, or $\frac{1}{8}$. Therefore, the difference required is $\frac{1}{8}$.

Rule.

Reduce the fractions, if necessary, to equivalent fractions having a common denominator, and write the difference of the numerators over the common denominator.

How may one fraction be subtracted from another? Repeat the Rule.

Examples.

What is the difference between

3. $\frac{5}{6}$ and $\frac{4}{5}$. Ans. $\frac{1}{30}$.
4. $\frac{8}{15}$ and $\frac{9}{20}$. Ans. $\frac{1}{12}$.
5. $\frac{3}{8}$ and $\frac{1}{2}$. Ans. $\frac{1}{8}$.
6. $\frac{7}{8}$ and $\frac{2}{8}$.
7. $\frac{1}{7}$ and $\frac{1}{11}$.
8. $\frac{15}{16}$ and $\frac{11}{12}$. Ans. $\frac{1}{48}$.
9. $\frac{11}{12}$ and $\frac{3}{4}$. Ans. $\frac{1}{6}$.
10. $\frac{15}{16}$ and $\frac{4}{7}$. Ans. $\frac{41}{112}$.
11. $\frac{16}{21}$ and $\frac{8}{15}$.
12. $\frac{3}{5}$ and $\frac{2}{7}$.

13. How much is $\frac{5}{9}$ greater than $\frac{4}{21}$? Ans. $\frac{23}{63}$.

14. Gave $\frac{5}{6}$ of a dollar for an arithmetic, and $\frac{2}{5}$ of a dollar for a slate; how much more did the arithmetic cost than the slate? Ans. $\frac{13}{30}$ of a dollar.

15. From 7 take $\frac{3}{4}$.

OPERATION.

$7 - \frac{3}{4} = \frac{28}{4} - \frac{3}{4} = \frac{25}{4} = 6\frac{1}{4}$, Ans.

Or, $7 - 1 = 6$; $1 = \frac{4}{4}$; $\frac{4}{4} - \frac{3}{4} = \frac{1}{4}$; $6 + \frac{1}{4} = 6\frac{1}{4}$, Ans.

Here, we change the whole number 7 to fourths, and subtract the $\frac{3}{4}$. We can, also, take 1 from the 7, leaving 6, reduce the 1 to fourths, subtract the $\frac{3}{4}$, and add the difference to the 6. The result will be the same by either process.

16. Subtract $2\frac{6}{7}$ from 5. Ans. $2\frac{1}{7}$.
17. Subtract $5\frac{1}{2}$ from 9. Ans. $3\frac{1}{2}$.
18. Subtract $6\frac{1}{2}$ from $12\frac{5}{12}$.

OPERATION.

$12\frac{5}{12} - 6\frac{1}{2} = \frac{149}{12} - \frac{78}{12} = \frac{71}{12} = 5\frac{11}{12}$, Ans.

Or, $12\frac{5}{12} - 6\frac{1}{2} = 11\frac{17}{12} - 6\frac{6}{12} = 5\frac{11}{12}$, Ans.

Explain the manner of working the 15th example.

Here, by the second process, the fractional parts having been reduced to fractions having a common denominator, and as we can not take 6 twelfths from 5 twelfths, we take 1 from 12, leaving 11, reduce the 1 to twelfths, making 12 twelfths, add them to the $\frac{5}{12}$, giving $\frac{17}{12}$, and, then, subtracting the $\frac{6}{12}$ from the $\frac{17}{12}$, and the 6 from the 11, we have $5\frac{11}{12}$, the difference required.

19. Subtract $2\frac{1}{2}$ from $5\frac{5}{8}$. Ans. $3\frac{1}{8}$.

20. Subtract $20\frac{1}{5}$ from $29\frac{3}{4}$

21. If from a cask containing $25\frac{3}{8}$ gallons of wine, $7\frac{2}{3}$ gallons leak out, how much remains in it?

22. Bought a lot of coal for $31\frac{7}{8}$ dollars, and sold it for $37\frac{9}{10}$ dollars; how much was gained?

Ans. $6\frac{1}{63}$ dollars.

23. Paid $10\frac{3}{10}$ dollars for a barrel of flour, and sold it for $1\frac{7}{12}$ dollars less than cost; how much was obtained for it? Ans. $8\frac{43}{60}$ dollars.

24. When a ton of coal, bought for $5\frac{9}{10}$ dollars, is sold for $7\frac{1}{4}$ dollars, how much is gained?

Ans. $1\frac{7}{20}$ dollars.

MULTIPLICATION.

158. A fraction is multiplied by multiplying its numerator, or by dividing its denominator.

For, let it be required to multiply $\frac{1}{8}$ by 4. Then 4 times 1 eighth are 4 eighths, which reduced is $\frac{1}{2}$. Now, if instead of taking the *number* of parts 4 times, we make the *size* of the parts 4 times as large, by dividing the denominator 8 by 4, we have, also, $\frac{1}{2}$ as before.

Explain the manner of working the 18th example. How is a fraction multiplied?

Case I.

159. To multiply a fraction by a whole number.

1. Multiply $\frac{4}{15}$ by 3.

OPERATION.

$$\frac{4}{15}\times 3=\frac{4\times 3}{15}=\frac{12}{15}=\frac{4}{5}, \text{ Ans.}$$

$$\text{Or, } \frac{4}{15}\times 3=\frac{4}{15\div 3}=\frac{4}{5}, \text{ Ans.}$$

3 times 4 *fifteenths*, or $\frac{4\times 3}{15}$, is $\frac{12}{15}$, which reduced is $\frac{4}{5}$; or, since dividing the denominator multiplies the fraction (Art. 158), we divide the denominator 15 by 3, and have $\frac{4}{15\div 3}$, or $\frac{4}{5}$, as before.

Rule.

Multiply the numerator by the whole number; or, *Divide the denominator by the whole number, when it can be done without a remainder.*

Examples.

Multiply

2. $\frac{7}{11}$ by 4. Ans. $2\frac{6}{11}$.
3. $\frac{8}{16}$ by 7. Ans. $3\frac{1}{2}$.
4. $\frac{3}{4}$ by 16. Ans. 12.
5. $\frac{3}{8}$ by 18.
6. $\frac{7}{12}$ by 11.
7. $\frac{15}{16}$ by 25. Ans. $23\frac{7}{16}$.
8. $\frac{3}{20}$ by 7. Ans. $1\frac{1}{20}$.
9. $\frac{18}{11}$ by 5. Ans. $8\frac{2}{11}$.
10. $\frac{19}{21}$ by 21.
11. $\frac{9}{13}$ by 26.

12. At $\frac{7}{8}$ of a dollar a bushel, what cost 12 bushels of corn? Ans. $10\frac{1}{2}$.

160. When the multiplicand is a *mixed number*, we may *multiply the fractional part and integral part separately, and add the products;* or, *reduce the mixed number to an improper fraction, and then multiply.*

Explain the operation. What is the Rule? How may we multiply when the multiplicand is a mixed number?

13. Required the product of $7\frac{3}{4}$ multiplied by 5.

OPERATION.

$$\begin{array}{lll} \frac{3}{4} \times 5 = \frac{3 \times 5}{4} = \frac{15}{4} = & 3\frac{3}{4} & \\ 7 \times 5 = & 35 & \\ \hline 7\frac{3}{4} \times 5 = & 38\frac{3}{4}, & \text{Ans.} \end{array}$$

$$\text{Or, } 7\tfrac{3}{4}, \times 5 = \frac{31 \times 5}{4} = \frac{155}{4} = 38\tfrac{3}{4}, \text{ Ans.}$$

14. Multiply $3\frac{2}{5}$ by 7. Ans. $23\frac{4}{5}$.

15. Multiply $8\frac{4}{7}$ by 6.

16. Multiply $10\frac{2}{9}$ by 9. Ans. 92.

17. Multiply $11\frac{3}{11}$ by 20.

18. If one man can build $15\frac{2}{3}$ rods of wall in a week, how many rods can 13 men build in the same time? Ans. $203\frac{2}{3}$.

Case II.

161. To multiply a whole number by a fraction.

Multiplying a number by a fraction is taking the *part* of it denoted by the fraction.

For, multiplying any number, as 6, by $\frac{1}{2}$, is taking 1 *half* of 6, by $\frac{1}{3}$ is taking 1 *third* of 6, by $\frac{2}{3}$ is taking 2 *thirds* of 6, etc.

1. Let it be required to multiply 16 by $\frac{3}{4}$.

OPERATION.

$$16 \times \tfrac{3}{4} = \frac{\overset{4}{\cancel{16}} \times 3}{\cancel{4}} = 12, \text{ Ans.}$$

Since 1 *fourth* of 16 is 4, 3 *fourths* of 16 is 3 times 4, or 12. Or 16 taken 3 times is 16 × 3, and taken 1 *fourth* as many times is $\frac{16 \times 3}{4}$, or 12, as before.

Explain the operation. What is multiplying a number by a fraction? Explain the operation.

Rule.

Multiply the whole number by the numerator, and divide the product by the denominator.

Examples.

Multiply

2. 48 by $\frac{2}{3}$.	Ans. 32.	7. 64 by $\frac{1}{7}$.	Ans. $9\frac{1}{7}$.
3. 33 by $\frac{3}{11}$.	Ans. 9.	8. 25 by $\frac{15}{16}$.	Ans. $23\frac{7}{16}$.
4. 16 by $\frac{4}{5}$.	Ans. $12\frac{4}{5}$.	9. 416 by $\frac{3}{8}$.	Ans. 156.
5. 18 by $\frac{5}{6}$.		10. 23 by $\frac{5}{12}$.	
6. 51 by $\frac{2}{7}$.		11. 101 by $\frac{2}{9}$.	

12. What cost 160 bushels of oats, at $\frac{5}{8}$ of a dollar a bushel? Ans. $ 100.

13. If an acre of land is worth $ 239 dollars, what is $\frac{2}{11}$ of it worth? Ans. $43\frac{5}{11}$.

162. When the multiplier is a *mixed number,* we may *multiply by the fractional and integral parts separately, and add the products; or, reduce the mixed number to an improper fraction, and then multiply.*

14. What is the product of 33 by $3\frac{2}{5}$?

OPERATION.

$$
\begin{array}{lcr}
33 \times \frac{2}{5} = \frac{33 \times 2}{5} = \frac{66}{5} & = & 13\frac{1}{5}. \\
33 \times 3 & & 99 \\
\hline
33 \times 3\frac{2}{5} = & & 112\frac{1}{5}, \text{ Ans.}
\end{array}
$$

$$\text{Or, } 33 \times 3\tfrac{2}{5} = 33 \times \tfrac{17}{5} = \tfrac{561}{5} = 112\tfrac{1}{5}, \text{ Ans.}$$

Repeat the Rule. When the multiplier is a mixed number, how do *you proceed?*

What is the product of

15. 12 by $9\frac{5}{8}$. Ans. $115\frac{1}{2}$.

16. 43 by $8\frac{5}{8}$.

17. 67 by $5\frac{2}{13}$. Ans. $345\frac{4}{13}$.

18. 112 by $12\frac{3}{8}$.

19. If a car-load of coal is worth $45, how much is $10\frac{4}{17}$ times that quantity worth? Ans. $460\frac{10}{17}$.

Case III.

163. To multiply a fraction by a fraction.

A fraction of a fraction, as $\frac{1}{3}$ of $\frac{3}{4}$, is called a *compound fraction*; and the multiplying of a fraction by a fraction is equivalent to finding the value of a compound fraction.

Thus, $\frac{3}{4} \times \frac{1}{2}$ is equivalent to taking $\frac{1}{2}$ of $\frac{3}{4}$.

1. Find the product of $\frac{4}{5}$ by $\frac{3}{8}$.

OPERATION.

$$\frac{4}{5} \times \frac{3}{8} = \frac{\cancel{4} \times 3}{5 \times \underset{2}{\cancel{8}}} = \frac{3}{10}, \text{ Ans.}$$

1 *eighth* of $\frac{4}{5}$ is $\frac{4}{5 \times 8}$, and 3 *eighths* of $\frac{4}{5}$ is $\frac{4 \times 3}{5 \times 8}$, or, by reducing, $\frac{3}{10}$.

Rule.

Multiply the numerators together for a new numerator, and the denominators for a new denominator.

What is a fraction of a fraction called? What is the multiplying of a fraction by a fraction equivalent to? Repeat the Rule.

Examples.

Multiply

2. $\frac{4}{5}$ by $\frac{2}{3}$. Ans. $\frac{8}{15}$.
3. $\frac{8}{11}$ by $\frac{1}{8}$. Ans. $\frac{1}{11}$.
4. $\frac{5}{3}$ by $\frac{2}{5}$. Ans. $\frac{2}{3}$.
5. $\frac{11}{12}$ by $\frac{6}{7}$.
6. $\frac{1}{4}$ of $\frac{2}{3}$ by $\frac{1}{2}$. Ans. $\frac{1}{12}$.
7. $\frac{7}{8}$ by $\frac{1}{5}$ of $\frac{3}{7}$. Ans. $\frac{3}{40}$.
8. $\frac{11}{14}$ by $\frac{7}{4} \times \frac{2}{11}$. Ans. $\frac{1}{4}$.
9. $\frac{9}{15} \times \frac{4}{3}$ by $\frac{2}{5}$.

10. What part of a ship is $\frac{1}{4}$ of $\frac{5}{8}$ of it? Ans. $\frac{5}{32}$.

11. What is the product of $\frac{3}{4}$, $\frac{2}{3}$, and $\frac{4}{5}$?

12. At $\frac{8}{25}$ of a dollar a yard, what will $\frac{5}{16}$ of a yard of cloth cost? Ans. $\frac{1}{10}$ of a dollar.

164. When there are *integral* or *mixed numbers* among the factors, *reduce them to improper fractions before multiplying.*

13. What is the product of $\frac{5}{7}$ of $4\frac{1}{10}$ by 7?

OPERATION.

$$\frac{5}{7} \times 4\frac{1}{10} \times 7 = \frac{5 \times 41 \times 7}{7 \times 10 \times 1} = \frac{41}{2} = 20\tfrac{1}{2}, \text{ Ans.}$$

14. What is the product of $7\frac{1}{2}$ by $9\frac{1}{4}$? Ans. $69\frac{3}{8}$.

15. What will $5\frac{1}{4}$ pounds of sugar cost, at $\frac{1}{6}$ of a dollar a pound? Ans. $\frac{7}{8}$ of a dollar.

16. What will $50\frac{5}{16}$ pounds of beef cost, at $\frac{4}{25}$ of a dollar a pound? Ans. $8\frac{1}{20}$.

17. What is the value of $\frac{3}{4}$ of $\frac{7}{8}$ of 9? Ans. $5\frac{29}{32}$.

18. When cloth is $\frac{7}{8}$ of a dollar a yard, what will $\frac{4}{7}$ of 12 yards cost? Ans. $ 6.

19. What is the value of $\frac{9}{10}$ of $\frac{5}{27}$ of $6\frac{3}{5}$?

20. At $\frac{9}{16}$ of a dollar a bushel, what cost $\frac{4}{18}$ of 32 bushels of apples? Ans. $ 4.

When there are integral or mixed numbers among the factors, how *do you proceed*?

DIVISION.

165. A fraction is divided by dividing its numerator, or by multiplying its denominator.

For, let it be required to divide $\frac{4}{8}$ by 4. Then, 1 fourth of 4 eighths, or $\frac{4 \div 4}{8}$, is $\frac{1}{8}$; or, if instead of finding 1 fourth of the *number* of parts, we make the *size* of the parts 1 fourth as large, by multiplying the denominator 8 by 4, we have $\frac{4}{8 \times 4}$, or, by reducing, $\frac{1}{8}$, as before.

Case I.

166. To divide a fraction by a whole number.

1. Let it be required to divide $\frac{18}{23}$ by 6.

OPERATION.

$$\frac{18}{23} \div 6 = \frac{18 \div 6}{23} = \frac{3}{23}, \text{ Ans.}$$

Since $\frac{18}{23}$ divided by 1 is $\frac{18}{23}$, $\frac{18}{23}$ divided by 6 is $\frac{1}{6}$ of $\frac{18}{23}$, which is $\frac{18 \div 6}{23}$, or $\frac{3}{23}$. Therefore, the quotient of $\frac{18}{23}$ divided by 6 is $\frac{3}{23}$.

2. What is the quotient of $\frac{1}{4}$ divided by 3?

OPERATION.

$$\frac{1}{4} \div 3 = \frac{1}{4 \times 3} = \frac{1}{12}, \text{ Ans.}$$

Since $\frac{1}{4}$ divided by 1 is $\frac{1}{4}$, $\frac{1}{4}$ divided by 3 is $\frac{1}{3}$ of $\frac{1}{4}$, which is $\frac{1}{4 \times 3}$, or $\frac{1}{12}$. Therefore, the quotient of $\frac{1}{4}$ divided by 3 is $\frac{1}{12}$.

Rule.

Divide the numerator by the whole number, when it can be done without a remainder; or,

Multiply the denominator by the whole number.

How is a fraction divided? Explain the operations. Repeat the Rule.

Examples.

Divide

3. $\frac{7}{8}$ by 4. Ans. $\frac{7}{32}$.
4. $\frac{21}{25}$ by 7. Ans. $\frac{3}{25}$.
5. $\frac{48}{37}$ by 12.
6. $\frac{7}{40}$ by 21.
7. $\frac{18}{20}$ by 27. Ans. $\frac{1}{30}$.
8. $\frac{24}{47}$ by 16. Ans. $\frac{3}{94}$.
9. $\frac{36}{65}$ by 18.
10. $\frac{45}{63}$ by 25.

11. How much is $\frac{8}{15}$ of $\frac{3}{4}$ divided by 10?

OPERATION.

$$\frac{\overset{2}{\cancel{8}} \times \cancel{3}}{\underset{5}{\cancel{15}} \times \cancel{4} \times \underset{5}{\cancel{10}}} = \frac{1}{25},\ \text{Ans.}$$

12. If 6 yards of cloth cost $\frac{15}{16}$ of a dollar, what will 1 yard cost? Ans. $\frac{5}{32}$ of a dollar.

13. When 11 pounds of coffee can be bought for $\frac{33}{16}$ of a dollar, how much is it a pound?
Ans. $\frac{3}{16}$ of a dollar.

167. When the dividend is a *mixed number, reduce it to an improper fraction before dividing.*

14. Find the quotient of $5\frac{1}{7}$ divided by 12.

OPERATION.

$$5\frac{1}{7} \div 12 = \frac{36}{7} \div 12 = \frac{\overset{3}{\cancel{36}}}{7 \times \cancel{12}} = \frac{3}{7},\ \text{Ans.}$$

When the dividend is a mixed number, how do you proceed?

15. $14\frac{3}{4}$ by 8. Ans. $1\frac{27}{32}$.
16. $21\frac{3}{7}$ by 25.
17. $4\frac{11}{13}$ by 9. Ans. $\frac{7}{13}$.
18. $30\frac{1}{4}$ by 22.

19. When 4 bushels of peaches cost $\$4\frac{1}{2}$, what is the cost of 1 bushel? Ans. $\$1\frac{1}{8}$.

20. If $\$28\frac{4}{5}$ be divided equally between 12 men, how much will each receive? Ans. $\$2\frac{2}{5}$.

21. How much is $\frac{5}{8}$ of 11 dollars divided by 22? Ans. $\frac{5}{16}$ of a dollar.

22. At \$15 a ton, how many tons of hay can be bought for $\$116\frac{1}{4}$? Ans. $7\frac{3}{4}$ tons.

Case II.

168. To divide a whole number by a fraction.

1. Let it be required to find the quotient of 12 divided by $\frac{1}{4}$.

OPERATION.

$$12 \div \frac{1}{4} = \frac{12 \times 4}{1} = 48, \text{ Ans.}$$

Since 12 divided by 1 is 12, 12 divided by $\frac{1}{4}$ is 4 times 12, or 48. Therefore, the quotient of 12 divided by $\frac{1}{4}$ is 48.

2. Find the quotient of 16 divided by $\frac{2}{5}$.

OPERATION.

$$16 \div \frac{2}{5} = \frac{\overset{8}{\cancel{16}} \times 5}{\cancel{2}} = 40, \text{ Ans.}$$

Since 16 divided by 1 is 16, 16 divided by $\frac{1}{5}$ is 5 times 16, and 16 divided by 2 fifths is $\frac{1}{2}$ of 5 times 16, which is $\frac{5}{2}$ times 16, or 40. Therefore, the quotient of 16 divided by $\frac{2}{5}$ is 40.

Explain the operation of Example 1. Of Example 2.

Rule.

Divide by the numerator of the divisor, and multiply by the denominator; or,

Invert the divisor, and proceed as in multiplication.

Examples.

Divide

3. 8 by $\frac{4}{5}$. Ans. 10.
4. 14 by $\frac{24}{25}$. Ans. $16\frac{2}{3}$.
5. 3 by $\frac{9}{10}$. Ans. $3\frac{1}{3}$.
6. 15 by $\frac{10}{11}$.
7. 75 by $\frac{3}{4}$. Ans. 100.
8. 5 by $\frac{7}{10}$. Ans. $7\frac{1}{7}$.
9. 27 by $\frac{5}{9}$. Ans. $48\frac{3}{5}$.
10. 34 by $\frac{17}{20}$.

11. At $\frac{3}{16}$ of a dollar a pound, how much coffee can be bought for $ 12? Ans. 64 pounds.

12. If it takes $\frac{2}{3}$ of a yard of cloth to make a vest, how many vests can be made from 6 yards. Ans. 9 vests.

169. When the divisor is a *mixed number, reduce it to an improper fraction, before dividing.*

13. Divide 12 by $1\frac{1}{2}$.

OPERATION.

$$12 \div 1\tfrac{1}{2} = 12 \times \tfrac{2}{3} = \frac{\overset{4}{\cancel{12}} \times 2}{\cancel{3}} = 8, \text{ Ans.}$$

14. 75 by $3\frac{2}{3}$. Ans. $20\frac{5}{11}$.
15. 18 by $1\frac{1}{4}$.
16. 92 by $5\frac{1}{9}$. Ans. 18.
17. 100 by $4\frac{7}{8}$.

Repeat the Rule. When the divisor is a mixed number, how do you *proceed?* Explain the operation.

18. At $6\frac{1}{4}$ cents a pound, how many pounds of rice can be bought for $ 1.50? Ans. 24 pounds.

19. How many bushels of wheat, at $ $2\frac{1}{6}$ a bushel, can be bought for $ 39? Ans. 18 bushels.

Case III.

170. To divide a fraction by a fraction.

1. Let it be required to divide $\frac{4}{5}$ by $\frac{2}{3}$.

OPERATION.

$$\frac{4}{5} \div \frac{2}{3} = \frac{4}{5} \times \frac{3}{2} = \frac{12}{10} = \frac{6}{5} = 1\frac{1}{5}, \text{ Ans.}$$

Since $\frac{4}{5}$ divided by 1 is $\frac{4}{5}$, $\frac{4}{5}$ divided by $\frac{1}{3}$ is 3 times $\frac{4}{5}$, and $\frac{4}{5}$ divided by 2 thirds is $\frac{1}{2}$ of 3 times $\frac{4}{5}$, which is $\frac{3}{2}$ times $\frac{4}{5}$, or $\frac{12}{10}$; or, by reducing, $1\frac{1}{5}$. Therefore, the quotient of $\frac{4}{5}$ divided by $\frac{2}{3}$ is $1\frac{1}{5}$.

Rule.

Divide by the numerator of the divisor, and multiply by the denominator; or,

Invert the divisor, and proceed as in multiplication.

If there are mixed numbers, reduce them to improper fractions.

Divide

2. $\frac{7}{8}$ by $\frac{1}{2}$. Ans. $1\frac{3}{4}$.
3. $\frac{2}{7}$ by $\frac{4}{11}$. Ans. $\frac{11}{14}$.
4. $\frac{5}{6}$ by $\frac{2}{9}$. Ans. $3\frac{3}{4}$.
5. $\frac{8}{9}$ by $\frac{4}{7}$.
6. $\frac{9}{13}$ by $\frac{3}{4}$.
7. $\frac{9}{30}$ by $4\frac{1}{2}$. Ans. $\frac{1}{15}$.
8. $3\frac{1}{6}$ by $9\frac{1}{2}$. Ans. $\frac{1}{3}$.
9. $\frac{7}{16}$ by $\frac{7}{8}$. Ans. $\frac{1}{2}$.
10. $9\frac{1}{2}$ by $\frac{7}{8}$.
11. $6\frac{1}{2}$ by $3\frac{1}{3}$.

Repeat the Rule.

12. How many pounds of tea, at $\frac{1}{4}$ of a dollar a pound, can be bought for \$$7\frac{1}{2}$? Ans. 30 pounds.

13. When for $\frac{7}{8}$ of a dollar $3\frac{1}{2}$ bushels of apples can be bought, what is the cost of 1 bushel?

Ans. $\frac{1}{4}$ of a dollar.

14. At $\frac{7}{8}$ of a dollar for $\frac{3}{4}$ of a yard of cloth, what is the price per yard? Ans. $1\frac{1}{6}$ dollars.

15. At $\frac{5}{8}$ of a dollar a gallon, how many gallons of kerosene can be bought for \$$5\frac{1}{2}$?

Ans. $8\frac{4}{5}$ gallons.

16. How many bottles, of $1\frac{1}{2}$ pints each, can be filled with $27\frac{3}{4}$ gallons of wine? Ans. 148 bottles.

Miscellaneous Examples.

1. Reduce $\frac{144}{240}$ to its lowest terms. Ans. $\frac{3}{5}$.
2. Reduce $\frac{23}{92}$ to its lowest terms.
3. Change \$31 to sixths of a dollar. Ans. \$$\frac{186}{6}$.
4. Reduce $19\frac{3}{7}$ to an improper fraction. Ans. $\frac{136}{7}$.
5. Change 22 to a fractional form. Ans. $\frac{22}{1}$.
6. Change 22 to a fraction whose denominator is 3.
7. Reduce $\frac{1}{3}$ and $\frac{1}{5}$ to fractions having a common denominator.
8. Add $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{9}$. Ans. $1\frac{29}{36}$.
9. Find the sum of $\frac{3}{5}$, $2\frac{3}{10}$, and $5\frac{1}{4}$. Ans. $8\frac{3}{20}$.
10. How much larger is $\frac{8}{15}$ than $\frac{9}{20}$? Ans. $\frac{1}{12}$
11. Find the difference between $\frac{10}{11}$ and $\frac{6}{7}$.
12. From $6\frac{11}{14}$ subtract $2\frac{11}{21}$. Ans. $4\frac{11}{42}$.
13. Change $\frac{131}{5}$ to a mixed number. Ans. $26\frac{1}{5}$.
14. Find the value of $\frac{13}{14} \times 6$. Ans. $5\frac{4}{7}$.
15. Multiply 51 by $\frac{5}{17}$. Ans. 15.

Review Questions.—What is an Integer? (127.) What is a *Prime Number?* (128.) What is a Composite Number? (129.)

16. How much is $\frac{8}{9}$ of $\frac{1}{2}$ of $\frac{6}{7}$? Ans. $\frac{8}{21}$.

17. Find the value of $\frac{3}{11} \times \frac{7}{4} \times \frac{8}{9}$.

18. Find the value of $\frac{2}{9} + \frac{5}{4} - 1\frac{1}{12}$. Ans. $\frac{7}{18}$.

19. Divide $\frac{17}{20}$ by 9. Ans. $\frac{17}{180}$.

20. Divide 26 by $\frac{13}{15}$. Ans. 30.

21. How many times is $\frac{7}{8}$ contained in 4?

22. What is the value of $\frac{12}{17} \div \frac{4}{5}$? Ans. $\frac{15}{17}$.

23. A merchant owning $\frac{3}{8}$ of a ship bought $\frac{5}{16}$ more, how much did he then own of it? Ans. $\frac{11}{16}$.

24. A farmer divided some hay into 20 equal parts, and gave 16 parts to his cows, and the rest to his sheep. What fraction of the whole did the sheep get? Ans. $\frac{1}{5}$.

25. What number is that to which if you add $3\frac{5}{9}$, the sum will be $16\frac{5}{12}$? Ans. $12\frac{31}{36}$.

26. If $\frac{3}{8}$ of a mill is worth $3740, what is the value of the whole? Ans. 9773\frac{1}{3}$.

27. When 13 tons of coal are sold for 71\frac{1}{2}$, what is the price per ton? Ans. $5\frac{1}{2}$.

28. Paid 10\frac{1}{2}$ for tea, at the rate of $\frac{7}{10}$ of a dollar a pound; how many pounds did I buy?
Ans. 15 pounds.

29. If the divisor is $\frac{5}{13}$, and the quotient $2\frac{1}{4}$, what is the dividend? Ans. $\frac{45}{52}$.

30. How many bushels of corn, at $\frac{9}{10}$ of a dollar a bushel, must be given for $1\frac{3}{5}$ of a ton of coal, at 4\frac{1}{2}$ a ton? Ans. 8.

31. When broadcloth is $4\frac{3}{8}$ dollars a yard, how many yards can be bought for 3 cords of wood, at 7 dollars a cord? Ans. $4\frac{4}{5}$ yards.

REVIEW QUESTIONS. — What is Analysis? (137.) What is a Fraction? (139.) A Common Fraction? (142.) The Terms of a Fraction? (143.)

Questions for Analysis.

1. If 1 barrel of flour cost $ 9, how much will $\frac{3}{4}$ of a barrel cost?

SOLUTION. *If* 1 *barrel cost* $ 9, $\frac{1}{4}$ *of a barrel will cost* $\frac{1}{4}$ *of* $ 9, *or* $ 2.25; *and* $\frac{3}{4}$ *will cost* 3 *times* $ 2.25, *or* $ 6.75. *Therefore, if* 1 *barrel of flour cost* $ 9, $\frac{3}{4}$ *of a barrel will cost* $ 6.75.

2. If 1 ton of coal be worth $ 5.25, what is $\frac{4}{5}$ of a ton worth? Ans. $ 4.20.

3. When $ 8.80 is paid for doing a certain amount of work, how much must be paid for $\frac{7}{11}$ as much? Ans. $ 5.60.

4. If an acre of land produce 1 T. 16 cwt. 18 lb. of hay, how much will $\frac{8}{9}$ of an acre produce? Ans. 1 T. 12 cwt. 16 lb.

5. If in a lot there are 15 A. 3 R. 28 P., what is the extent of $\frac{5}{7}$ of it? Ans. 11 A. 1 R. 20 P.

6. If $\frac{3}{4}$ of a barrel of flour cost $ 6.75, what will 1 barrel cost?

SOLUTION. *If* $\frac{3}{4}$ *of a barrel cost* $ 6.75, $\frac{1}{4}$ *of a barrel will cost* $\frac{1}{3}$ *of* $ 6.75, *or* $ 2.25, *and* $\frac{4}{4}$, *or* 1 *barrel, will cost* 4 *times* $ 2.25, *or* $ 9. *Therefore, if* $\frac{3}{4}$ *of a barrel of flour cost* $ 6.75, 1 *barrel will cost* $ 9.

7. If $\frac{7}{8}$ of a yard of cloth cost $ 3.78, what will a whole yard cost? Ans. $ 4.32.

8. Paid $ 156 for a horse, which was $\frac{13}{15}$ as much as I paid for a wagon; how much did I pay for the wagon? Ans. $180.

9. A farmer sold 31 bu. 3 pk. 6 qt. of grain, which was $\frac{7}{8}$ of all he had; how much had he? Ans. 36 bu. 2 pk.

10. If 11 A. 1 R. 15 P. is $\frac{5}{7}$ of a certain field, what is its entire extent? Ans. 15 A. 3 R. 21 P.

11. If 4 tons of hay cost \$ 52, what will $3\frac{2}{13}$ tons cost?

SOLUTION. *If* 4 *tons cost* \$ 52, 1 *ton will cost* $\frac{1}{4}$ *of* \$ 52, *or* \$ 13. $3\frac{2}{13}$ *tons are* $\frac{41}{13}$ *tons; and if* 1 *ton cost* \$ 13, $\frac{1}{13}$ *will cost* $\frac{1}{13}$ *of* \$ 13, *or* \$ 1, *and* $\frac{41}{13}$ *will cost* 41 *times* \$ 1, *or* \$ 41. *Therefore, if* 4 *tons of hay cost* \$ 52, $3\frac{2}{13}$ *tons will cost* \$ 41.

12. When \$ 60 are paid for 12 yards of broadcloth, how much must be paid for $9\frac{7}{8}$ yards? Ans. \$ 49.37$\frac{1}{2}$.

13. If $3\frac{2}{13}$ tons of hay cost \$ 41, what will 4 tons cost?

14. If 9 bushels of wheat are worth \$ 19.35, what are $10\frac{3}{5}$ bushels worth? Ans. \$ 22.79.

15. If 17 bushels of corn cost \$ 16.32, what will $15\frac{3}{4}$ bushels cost? Ans. \$15.12.

16. If $\frac{2}{3}$ of an acre of land is worth \$ 60, what is $\frac{5}{6}$ of an acre worth?

SOLUTION. *If* $\frac{2}{3}$ *of an acre is worth* \$ 60, $\frac{1}{3}$ *is worth* $\frac{1}{2}$ *of* \$ 60, *or* \$ 30, *and* $\frac{3}{3}$, *or* 1 *acre, is worth* 3 *times* \$ 30, *or* \$ 90; *and if* 1 *acre is worth* \$ 90, $\frac{1}{6}$ *of an acre is worth* $\frac{1}{6}$ *of* \$ 90, *or* \$ 15, *and* $\frac{5}{6}$ *is worth* 5 *times* \$ 15, *or* \$ 75. *Therefore, if* $\frac{2}{3}$ *of an acre is worth* \$ 60, $\frac{5}{6}$ *of an acre is worth* \$ 75.

17. If $\frac{3}{8}$ of a cord of wood cost \$ 2.85, what will $\frac{3}{4}$ of a cord cost? Ans. \$ 5.70.

18. When $\frac{3}{4}$ of a pound of tea can be bought for \$ 0.54, what will $4\frac{1}{3}$ pounds cost? Ans. \$ 3.12.

19. If $4\frac{1}{2}$ pounds of butter are worth \$ 1.44, what are $11\frac{7}{16}$ pounds worth? Ans. 3.66.

DECIMAL FRACTIONS.

171. If a unit be divided into ten equal parts, each of these parts will be 1 *tenth*.

If each tenth be divided into ten equal parts, each part will be $\frac{1}{10}$ of $\frac{1}{10}$, or 1 *hundredth*.

If each hundredth be divided into ten equal parts, each part will be $\frac{1}{10}$ of $\frac{1}{100}$, or 1 *thousandth*.

In like manner, we may, by dividing by ten, continue to obtain fractions, each of whose values is *one tenth* of the fraction preceding it; such fractions are called *Decimal Fractions*.

172. **A Decimal Fraction**, or a **Decimal**, is a fraction whose unit is divided into *tenths*, *hundredths*, *thousandths*, etc.

NOTATION AND NUMERATION.

173. A decimal fraction, by means of the decimal point (.), is written without the denominator, and the value of the decimal figures determined by the place of each with reference to the decimal point.

The first order to the right of the decimal point is that of *tenths*. Thus,

$\frac{1}{10}$ may be written .1, and read 1 tenth;
$\frac{2}{10}$ " " .2, " 2 tenths;
$\frac{3}{10}$ " " .3, " 3 tenths;

and so on.

What is a Decimal Fraction? How is it written? What is the first order to the right of the point?

The second order to the right of the decimal point is that of *hundredths.* Thus,

$\frac{1}{100}$ may be written .01, and read 1 hundredth ;
$\frac{2}{100}$ " " .02, " 2 hundredths ;
$\frac{3}{100}$ " " .03, " 3 hundredths ;
and so on.

The third order to the right of the decimal point is that of *thousandths.* Thus,

$\frac{1}{1000}$ may be written .001, and read 1 thousandth ;
$\frac{2}{1000}$ " " .002, " 2 thousandths ;
$\frac{3}{1000}$ " " .003, " 3 thousandths ;
and so on.

The fourth order to the right is *ten-thousandths;* the fifth, *hundred-thousandths,* and so on.

174. Since decimals of the first order are *tenths,* of the second order *hundredths,* of the third order *thousandths,* etc., it follows that

The denominator of a decimal fraction is 1 *with as many ciphers annexed as the numerator has decimal places.*

175. Since each lower order is one tenth of the next higher, it follows, in decimals, as in simple whole numbers (Art. 33), that

Ten of any lower order are always equal to one of the next higher.

Hence integers and decimals may be expressed together as a mixed number, by placing the decimal point between them.

What is the second order to the right of the point? The third? The fourth?

176. The orders of decimals are named from the decimal point to the right; thus,

Tenths, hundredths, thousandths, ten-thousandths, hundred-thousandths, millionths, etc. Hence, the

Table.

Integers.							Decimals.					
Hundred-thousands.	Ten-thousands.	Thousands.	Hundreds.	Tens.	Units.		Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.
3	4	6	7	8	2	.	5	0	9	1	3	2,

where the mixed number is read, three hundred forty-six thousand seven hundred eighty-two *units,* and five hundred nine thousand one hundred thirty-two *millionths.*

177. In United States Money, the dollar is the unit, and cents and mills are decimal parts of a dollar.

Thus, $5.25 is the same as 5 dollars and 25 hundredths of a dollar; and $7.125 is the same as 7 dollars and 125 thousandths of a dollar.

How are the orders named? Give the mixed number expressed in *the Table.* How may cents and mills be regarded?

178. To read decimals expressed by figures.

1. Let it be required to read the decimal .035.

SOLUTION. The figures are 0 tenth, 3 hundredths, and 5 thousandths; or, since 3 hundredths equal 30 thousandths, and 30 thousandths plus 5 thousandths, equal 35 thousandths, we have as the value expressed, 35 thousandths. Therefore, .035 is read *thirty-five thousandths.*

Rule.

Read the decimal as a whole number, giving it the name of the right-hand order.

Examples.

Write and read the following decimals:

2. .15
3. .05
4. .005
5. .125
6. .5
7. .75
8. .603
9. .1325
10. .506
11. .7153
12. .0174
13. .006501

179. When the decimal is part of a mixed number, we may *read the integral part as expressing units.*

Thus, 16.45 may be read, *sixteen units and forty-five hundredths.*

Write and read the following mixed numbers: —

14. 4.026.
15. 21.006.
16. 15.15.
17. 35.1675.
18. 91.289.
19. 102.05.
20. 5067.5.
21. 183.57.
22. 4.30068.

Repeat the Rule.

180. To write decimals in figures.

1. Let it be required to express in figures thirteen thousandths.

Solution. Since thirteen thousandths equal 1 hundredth and 3 thousandths, we write 3 in the order of thousandths, 1 in the order of hundredths, and, as there are no tenths, 0 in the order of tenths; and, prefixing the decimal point, we have .013.

Rule.

Write the figures at the right of the decimal point, each in its proper order, noting vacant orders, if any, by ciphers.

Examples.

Write in the decimal form:—

2. $\frac{7}{10}$. Ans. .7.
3. $\frac{3}{100}$ Ans. .03.
4. $\frac{5}{1000}$.
5. $\frac{65}{1000}$.
6. $\frac{106}{1000}$. Ans. .106.
7. $\frac{7}{1000}$. Ans. .007.
8. $\frac{14}{10000}$.
9. $\frac{1068}{10000}$
10. $7\frac{16}{100}$. Ans. 7.16
11. $19\frac{9}{1000}$. Ans. 19.009.
12. $103\frac{21}{100}$.
13. $162\frac{121}{10000}$.

14. Three hundred six thousandths. Ans. .306.

15. Twenty-three ten thousandths. Ans. .0023.

16. Thirty-six units, and five tenths. Ans. 36.5.

17. One hundred one units, and sixty-five hundredths. Ans. 101.65.

18. Sixteen units, and sixteen thousandths. Ans. 16.016.

19. Three hundred twenty-five ten thousandths.

20. Five units, and three hundred thousandths. 5.00003.

Repeat the Rule.

REDUCTION.

181. *Annexing a cipher to a decimal does not alter the value of the decimal.*

For, the order of the significant figures of the decimal is not changed Thus, .3, or .30, is the same as $\frac{3}{10}$.

182. Hence, to change decimals having different denominators, to equivalent decimals having a common denominator,

Make each decimal have the same number of decimal places, by annexing ciphers.

Exercises.

Change to equivalent fractions having a common denominator,

1. .7 and .05. Ans. .70 and .05.
2. .06 and .103.
3. .5216 and .16.
4. .0031 and .316. Ans. .0031 and .3160.
5. .161 and .01.
6. .8 and .09163.

Case I.

183. To reduce a decimal to a common fraction.

1. Let it be required to change .35 to a common fraction.

OPERATION.

$.35 = \frac{35}{100} = \frac{7}{20}$, Ans.

Removing the decimal point and writing the denominator (Art. 174), we have $\frac{35}{100}$, which reduced is $\frac{7}{20}$. Therefore, .35 is equal to $\frac{7}{20}$.

What effect has the annexing of a cipher to a decimal? How do you change decimals having different denominators to equivalent decimals *having a* common denominator?

Rule.

Write the denominator to the decimal, omit the decimal point, and reduce the common fraction to its lowest terms.

Examples.

Reduce to common fractions:—

2. .75. Ans. $\frac{3}{4}$.
3. .375. Ans. $\frac{3}{8}$.
4. .06.
5. .008. Ans. $\frac{1}{125}$.
6. .0075. Ans. $\frac{3}{400}$.
7. .0625.

Case II.

184. To reduce a common fraction to a decimal.

1. Let it be required to change $\frac{3}{4}$ to a decimal fraction.

OPERATION.

$$4\overline{)\,3.00}$$
$$.75, \text{ Ans.}$$

$\frac{3}{4}$ equals $3 \div 4$. Since we can not divide 3 by 4, we reduce 3 to tenths, and have 30 tenths; $\frac{1}{4}$ of 30 tenths is 7 tenths, with a remainder of 2 tenths; we write the 7 tenths. The 2 tenths we reduce to hundredths, and have 20 hundredths; $\frac{1}{4}$ of 20 hundredths is 5 hundredths, which we write. Therefore, $\frac{3}{4}$ equals .75.

Rule.

Annex ciphers to the numerator, divide by the denominator, and point off as many places for decimals as there were ciphers annexed.

Repeat the Rule for reducing a decimal to a common fraction. *Repeat the* Rule for reducing a common fraction to a decimal.

Examples.

Reduce to decimals: —

2. $\frac{5}{8}$. Ans. .625.
3. $\frac{1}{25}$ Ans. .04.
4. $\frac{3}{8}$.
5. $\frac{17}{20}$.
6. $\frac{3}{125}$. Ans. .024.
7. $\frac{3}{5}$. Ans. 6.
8. $\frac{3}{75}$.
9. $\frac{9}{16}$.

185. When a common fraction cannot be exactly expressed in decimals, the division may be carried to a sufficient degree of exactness, and the sign + annexed to the result to indicate its incompleteness.

10. Reduce $\frac{4}{9}$ to a decimal of three places. Ans. .444+.
11. Reduce $\frac{6}{7}$ to a decimal of four places. Ans. .8571+.
12. Reduce $\frac{5}{12}$ to a decimal of four places. Ans. .4166+.

ADDITION.

186. Since ten of any order of decimals make one of the order next higher (175), decimals may be added in the same manner as simple whole numbers. Hence, the

Rule.

Write the numbers so that figures of the same order shall stand in the same column, add as in whole numbers, and place the decimal point in the amount under those in the numbers added.

Why may decimals be added the same as simple whole numbers? Repeat the Rule.

Examples.

	(1.)	(2.)	(3.)
Add	2.35	41.144	.0632
	42.08	13.755	.456
	9.52	2.50	.81
	18.005	25.3	3.08
Ans.	71.955	82.699	4.4092

4. Add 6.051, .095, 31.6, and 1.0075.

5. Add 13.015, .9031, 12.186, and .09.

6. Add 120.16, .003, 1401.7, 1.5, and 19.8.

7. Add 7 units and 8 thousandths, 28 units and 16 hundredths, 56 units and 7 tenths. Ans. 91.868.

8. A merchant bought four lots of goods; for the first he paid $69.125; for the second, $193.3; for the third, $1008.56; and for the fourth, $752.375. How much did he pay for the whole? Ans. $2023.36.

SUBTRACTION.

187. Since ten units of any order in decimals make one of the order next higher (175), decimals may be subtracted in the same manner as simple whole numbers. Hence, the

Rule.

Write the less number under the greater, so that figures of the same order shall stand in the same column; subtract as in whole numbers, and place the decimal point in the difference under those in the numbers above.

Why may decimals be subtracted as in whole numbers? Repeat the *Rule.*

Examples.

	(1.)	(2.)	(3.)
From	17.083	97.031	106.57
Take	9.36	18.346	41.139
Ans.	7.723		65.431

Here, in Example 3, as there are no thousandths in the minuend to subtract from, we consider that order in the minuend as filled by 0, since annexing a cipher to a decimal does not alter its value (Art. 181.)

4. Subtract 5.91 from 13.675. Ans. 7.765.

5. Subtract 18.16 from 19.5. Ans. 1.34.

6. From 37 dollars take 31.375 dollars. Ans. $ 5.625.

7. Bought 160 acres of land, and sold from it 141.125 acres ; how much then had I left ? Ans. 18.875 acres.

8. From 15 take 15 hundredths. Ans. 14.85.

9. A farmer owing 78 hundredths of a farm, sold 725 thousandths of it ; how much of it had he left ? Ans. .055.

10. From 10 take 1 ten-thousandth. Ans. 9.9999.

11. Subtract 139.216 from 400.95. Ans. 261.734.

12. What is the difference between 64.075 and .195326 ?

13. From 107 take .0007. Ans. 106.9993.

14. From forty-three units and seventy-five thousandths, take thirty-five units and sixty-seven hundred thousandths. Ans. 8.07433.

MULTIPLICATION.

188. *Each removal of the decimal point one place toward the right multiplies by* 10.

For, each figure is made by the removal to denote units of an order next higher; hence the value expressed is made tenfold (Art. 33).

Thus, .08 × 10 = .8; .8 × 10 = 8; .306 × 100 = 30.6.

189. *Each removal of the decimal point one place toward the left divides by* 10.

For, each figure is made by the removal to denote units of an order next lower; hence, the value expressed is made one tenth as much as it was.

Thus, 6.5 ÷ 10 = .65; .65 ÷ 10 = .065; 73.4 ÷ 100 = .734.

190. To multiply when one or both of the factors are decimals.

1. Multiply 3.16 by .4.

OPERATION.

3.16
.4
Ans. 1.264

3.16 multiplied by 4 gives 12.64; but as the multiplier is .4, the product must be only a tenth as large, and 12.64 divided by 10, by removing the decimal point one place to the left (Art. 189), gives 1.264. Therefore, the product of 3.16 by .4 is 1.264.

What effect has the removal of a decimal point one place toward the *right?* One place toward the left? Explain the operation.

Rule.

Multiply as in whole numbers, and point off as many figures for decimals in the product as there are in both multiplicand and multiplier, supplying the deficiency, if any, by prefixing ciphers; or,

If the multiplier is a decimal, multiply by its numerator, and divide by its denominator.

When the multiplier is 10, 100, 1000, etc., remove the point to the right as many places as there are ciphers on the right of the multiplier (Art. 188).

Examples.

	(2.)	(3.)	(4.)
Multiply	.123	3162	.7869
By	.6	.08	100
Ans.	.0738	252.96	78.69

Multiply

5. 3.21 by 2.31. Ans. 7.4151.
6. 1000 by .7 Ans. 700.
7. .106 by .35.
8. 3.18 by 41.7.
9. .001 by .01. Ans. .00001.
10. 78 by .42. Ans. 32.76.
11. 9.16 by 10.
12. 65 by .65.
13. .036 by .08. Ans. .00288.
14. 75 by .8. Ans. 60.
15. .009 by .009.
16. .063 by 1000.

17. What cost .65 of an acre of land at $84 an acre? Ans. $54.60.

18. Multiply 12 hundredths by 8 thousandths.

Repeat the Rule. How do you multiply by 10, 100, 1,000, etc

19. What cost 18.75 barrels of flour, at $6.75 a barrel? Ans. $126.5625.

20. Find the value .1 × .1 × .1. Ans. .001.

21. Find the value of .0002 × 3.01. Ans. .000602.

22. What cost 18.25 yards of cambric, at $0.125 a yard? Ans. $2.28125.

23. When $14.56 is the cost of a load of coal, what is the cost of 2.25 loads? Ans. $32.76.

24. Multiply thirty-six units and forty-eight hundredths, by four hundred seventy-five thousandths.

DIVISION.

191. Since the dividend is equal to the product of the divisor and quotient, it must have as many decimal figures as there are in both divisor and quotient (Art. 190). Hence,

The quotient must have as many decimal figures as the number of decimal figures in the dividend exceeds that in the divisor.

192. To divide when the divisor, or dividend, or both, are decimals.

1. Divide 1.264 by .4.

OPERATION.

.4) 1.264

Ans. 3.16

1.264 divided by 4 gives .316; but, as the divisor is .4, the quotient must be ten times as large, and .316 multiplied by 10, by removing the decimal point one place to the right (Art. 188), gives 3.16. Therefore, the quotient of 1.264 divided by .4 is 3.16.

What is the dividend equal to? How many decimal figures must the quotient have?

2. Divide .00115 by .05.

OPERATION.

.05) .00115

.023

.00115 divided by 5 gives .00023; but, as the divisor is .05, the quotient must be one hundred times as large, and .00023 multiplied by 100, by removing the point two places to the right (Art. 188), gives .023. Therefore, the quotient of .00115 divided by .05 is .023.

Rule.

Divide as in whole numbers, and point off as many figures for decimals in the quotient as the number of decimal figures in the dividend exceeds that in the divisor, supplying the deficiency, if any, by prefixing ciphers; or,

If the divisor is a decimal, divide by its numerator, and multiply by its denominator.

When the divisor is 10, 100, 1000, etc., remove the point to the left as many places as there are ciphers in the divisor (Art. 189.)

Examples.

Divide

3. 8.05 by .23. Ans. 35.
4. 8.05 by 2.3. Ans. 3.5.
5. 8.05 by 23. Ans. .35.
6. .805 by 2.3.
7. 80.5 by 2.3.
8. 9.065 by .049. Ans. 185.
9. 1.31 by 100. Ans. .0131.
10. 15.625 by 2.5. Ans. 6.25.
11. 1.31 by 10.
12. 1.31 by 1000.

Explain the operation. Repeat the Rule. How do you proceed when the divisor is 10, 100, 1000, etc.?

193. When the number of decimal figures in the divisor exceeds that in the dividend, annex to the dividend as many decimal ciphers, at least, as will make the number of decimal figures in both equal.

13. Divide 805 by 2.3. 14. Divide 1 by .13.

OPERATION.

```
2.3 ) 805.0 ( 350 Ans.
      69
      ---
      115
      115
      ---
        0
```

OPERATION.

```
.13 ) 1.0000 ( 7.69+ Ans.
        91
       ----
         90
         78
         --
        120
        117
        ---
          3
```

Here, in Example 13, the division does not terminate. In such cases, after carrying it to a sufficient exactness, denote the incompleteness by annexing the sign +.

15. Divide 2.117 by .0073. Ans. 290.
16. Divide 8 by .002. Ans. 4000.
17. Divide 7.43 by .0079. Ans. 940.5063+.
18. Divide 89 by .003. Ans. 29666.66+.
19. Find the quotient of 80.3 divided by .22.
20. Find the quotient of 80.3 divided by 2.2. Ans. 36.5.

How do you proceed when the number of decimal figures in the divisor exceeds that in the dividend?

21. If 3.5 acres produce 142.1 bushels of corn, what is the yield per acre? Ans. 40.6 bu.

22. How many yards of cotton cloth, at .125 dollars per yard, can be bought for $46.95?

Ans. 375.6 yd.

23. At .75 dollars a day, how many days must a boy work to earn $141? Ans. 188 days.

24. If you can walk 12.5 miles in one day, how many days will it take you to walk 102.5?

25. Bought 30 pounds of butter for $5.85; how much was that a pound? Ans. $0.195.

26. Divide one hundredth by one ten-thousandth.

Ans. 100.

Miscellaneous Examples.

1. Add seven hundred forty-three thousandths, seven hundredths, five units and sixteen ten-thousandths.

Ans. 5.8146.

2. Add eighty-seven units and twenty-nine hundredths, seventy-three units and forty-seven thousandths, three thousand five units and one hundred six ten-thousandths, twenty-eight units and three hundredths, one unit and five thousandths.

Ans. 3194.3826.

3. Reduce $5\frac{3}{16}$ to the decimal form. Ans. 5.1875.

4. Reduce $\frac{17}{25}$ and $\frac{13}{250}$ to decimals having a common denominator. Ans. .680 and .052.

5. From 100 subtract 101 thousandths.

Ans. 99.899.

Review Questions.—What is a Unit? (1.) What is the Unit of a Fraction? *Ans.* The unit or single thing divided into equal parts.

6. From eighteen units and three hundred sixty-five thousandths, take eighteen hundred sixty-five ten thousandths. Ans. 18.1785.

7. Change .68 to an equivalent common fraction. Ans. $\frac{17}{25}$.

8. Change 5.25 to an equivalent common fraction. Ans. $\frac{21}{4}$.

9. Multiply one hundred units and thirty-seven hundredths by nine tenths. Ans. 900.333.

10. What common fraction is equal to the sum of .125 and .525? Ans. $\frac{13}{20}$.

11. Multiply .0235 by 8.08. Ans. .18988.

12. Divide .0625 by 2.5. Ans. .025.

13. At $1.25 a pair, how many pairs of shoes can be bought with $45? Ans. 36 pairs.

14. If an acre produces 16.25 bushels of wheat to an acre, how much will 3.05 acres produce?

15. If $18.5625 is paid for 148.5 yards of calico, how much is paid per yard? Ans. $.125.

16. If 3.5 cords of wood cost $22.4, how much will 4.25 cords cost? Ans. $27.20.

17. When $107.525 is paid for 11½ barrels of flour, how much must be paid for 5.8 barrels? Ans. $54.23.

18. At .375 of a dollar a bushel, how many bushels of potatoes can be bought for $500? Ans. 1333.33+ bushels.

19. If 51 pairs of shoes cost $ 133.875, how much will 67 pairs cost? Ans. $ 175.875.

REVIEW QUESTIONS.—What is a Decimal Fraction? (172). What the Denominator of a Decimal Fraction? (174.) How many decimal figures has the product in Multiplication of Decimals? (190.) The *quotient in Division* of Decimals? (192.)

PERCENTAGE.

194. Percentage is the process of computing in hundredths.

The term *per cent*, from *per*, by or on, and *centum*, hundred, means *by* or *on a hundred;* and 1 per cent. is 1 hundredth ; 2 per cent., 2 hundredths ; etc.

195. Any per cent. may be expressed in the form of a *decimal* or of a *common fraction*. Thus,

1 per cent.	is written	.01,	or	$\frac{1}{100}$
5 per cent.	“	.05,	“	$\frac{5}{100}$
10 per cent.	“	.10,	“	$\frac{10}{100}$
25 per cent.	“	.25,	“	$\frac{25}{100}$
100 per cent.	“	1.00,	“	$\frac{100}{100}$
125 per cent.	“	1.25,	“	$\frac{125}{100}$

$\frac{1}{2}$ per cent. is .00$\frac{1}{2}$ or .005 ; $\frac{1}{4}$ per cent. is .00$\frac{1}{4}$, or .0025 ; $\frac{1}{8}$ per cent. is .00$\frac{1}{8}$, or .00125 ; etc.

The *sign* % is used for the word per cent.

The *percentage* of a number is such a part of it as is denoted by the per cent.

Exercises.

Write decimally :—

1. 4 %. Ans. .04.
2. 6 %. Ans. .06.
3. 9 %.
4. 5$\frac{1}{2}$ %. Ans. .055.
5. 2$\frac{1}{4}$ %. Ans. .0225.
6. $\frac{1}{10}$ %.
7. 107 %. Ans. 1.07.
8. 160 %. Ans. 1.60.
9. 115 %.

What is Percentage? What is the term per cent. from? What does it mean? How may per cent. be expressed?

Write in form of a common fraction in its lowest terms : —

10. 8 %. Ans. $\frac{2}{25}$.
11. 20 %. Ans. $\frac{1}{5}$.
12. 50 %.
13. $37\frac{1}{2}$ %. Ans. $\frac{3}{8}$.
14. 75 %. Ans. $\frac{3}{4}$.
15. 90 %.
16. 110 %. Ans. $\frac{11}{10}$.
17. 150 %. Ans. $\frac{3}{2}$.
18. 125 %.

Mental Exercises.

1. What is 5 per cent. of 10 bushels ?

SOLUTION. *Since* 1 *per cent. of* 10 *bushels is* 1 *hundredth of* 10 *bushels,* 5 *per cent. will be* 5 *hundredths of* 10 *bushels, which is* 50 *hundredths of* 1 *bushel, or* $\frac{1}{2}$ *of* 1 *bushel. Therefore,* 5 *per cent. of* 10 *bushels is* $\frac{1}{2}$ *of a bushel.*

2. How much is 9 per cent. of 10 yards ?
3. How much is 6 per cent. of 12 dollars ?
4. If you earn $24, and spend 4 per cent. of it, how much will you have left ?
5. If I receive $4 profit on $100 of investment, how much is that on $7 ?

SOLUTION. *If the profit on* $100 *is* $4, *the profit on* 1 *dollar will be* 1 *hundredth of* $4, *and on* $7 *it will be* 7 *times* 1 *hundredth of* $4, *which is* $$\frac{28}{100}$, *or* $.28. *Therefore, if the profit on* $100 *is* $4, *on* $7 *it is* $.28.

REVIEW QUESTIONS. — What is a Decimal Fraction ? (172.) What is the denominator of a Decimal Fraction ? (174.)

6. If I lose 11 yards of cloth on 100 yards, how much do I lose on 10 yards?

7. When I gain $6 on $100, how much do I gain on $30?

8. Bought some articles for $100, and sold them for $112; how much did I gain on $5?

9. What per cent. of 25 is 5?

SOLUTION. *Since 25 is* 100 *per cent. of itself,* 1 *is* $\frac{1}{25}$ *of* 100 *per cent., and* 5 *is* 5 *times* $\frac{1}{25}$ *of* 100 *per cent., or* $\frac{5}{25}$ *of* 100 *per cent., which is* $\frac{1}{5}$ *of* 100 *per cent., or* 20 *per cent. Therefore,* 5 *is* 20 *per cent. of* 25.

10. What per cent. of 50 tons is 20 tons? Of 50 tons is 30 tons?

11. What per cent. of $35 is $7? Of $40 is $30?

12. If the cost of raising corn is 9 bushels to every 27 grown, what is the cost per cent.?

Written Exercises

196. To find any per cent. of a number.

1. What is 3 per cent. of $220?

OPERATION.

$ 220
.03
Ans. $ 6.60

Since 1 per cent. of $220 is 1 hundredth of $220, 3 per cent. will be 3 hundredths of $220, which is $220 × .03, or $6.60. Therefore, 3 per cent. of $220 is $6.60.

Rule.

Multiply the given number by the number of per cent., expressed decimally.

Explain the operation. What is the Rule?

Examples.

What is

2. 5 % of \$72. Ans. \$3.60.
3. 8 % of \$90. Ans. \$7.20.
4. 6 % of \$112.
5. 9 % of \$38.
6. 3½ % of 164 yd. Ans. 5.74 yd.
7. 5¼ % of 8.75 bu. Ans. .459 + bu.
8. 7 % of 330 lb.
9. 20 % of 416 cu. ft.

10. A farmer raised 160 bushels of apples, and sold 80 per cent. of the whole; how many did he sell? Ans. 128 bu.

11. Of \$500, I have invested in the United States Stocks 40 per cent., and the balance in land; how much have I invested in stock, and how much in land? Ans. stock \$200; land \$300.

12. From a deposit of \$1800 in a bank, I have taken out 62 per cent., how much remains? Ans. \$684.

197. To find what per cent. one number is of another.

1. What per cent. of 16 yards is 4 yards?

OPERATION.

$$\frac{\cancel{4} \times 100}{\underset{4}{\cancel{16}}} = 25\text{, Ans.}$$

Since 16 is 100 per cent. of itself, 1 is $\frac{1}{16}$ of 100 per cent., and 4 is 4 times $\frac{1}{16}$, or $\frac{4}{16}$, of 100 per cent., which is $\frac{1}{4}$ of 100 per cent., or 25 per cent. Therefore, 4 yards is 25 per cent. of 16 yards.

Explain the operation.

Rule.

Multiply the number denoting the percentage by 100, *and divide by the number on which the percentage is reckoned.*

Examples.

What per cent. is

2. 6 of 50 ?
Ans. 12 per cent.
3. 7 of 140 ?
Ans. 5 per cent.
4. 9 of 150 ?
5. 15 of 90 ?
6. 7 lb. of 56 lb. ?
Ans. 12½ per cent.
7. $ 12 of $ 60 ?
Ans. 20 per cent.
8. 25 men of 225 men ?
9. 21 miles of 280 miles ?

10. If American gold coin is 9 parts pure gold to 1 part alloy, what per cent. is alloy ?

Ans. 10 per cent.

11. If a miller takes 4 quarts toll for grinding 32 quarts, what per cent. is the toll ?

Ans. 12½ per cent.

12. A farmer's flock of 120 sheep was increased to 156 ; what was the increase per cent. ?

Ans. 30 per cent.

13. I have a farm of 160 acres : 120 acres of it is under cultivation ; what part is that of the whole ?

Ans. 75 per cent.

14. John is 12 years old, and his father 54 years ; what per cent. is John's age of that of his father ?

Ans. $22\frac{2}{9}$ per cent.

Repeat the Rule.

COMMISSION AND BROKERAGE.

198. A **Commission Merchant,** or **Agent,** is a person who transacts business for another.

199. A **Broker** is a person who makes or concludes bargains for a fee, or rate per cent.

200. Commission is the percentage paid a commission merchant, or agent, for the transaction of business.

201. Brokerage is the percentage paid a broker for the transaction of business.

202. Commission and Brokerage are reckoned in the same manner.

203. To find the commission or brokerage.

1. A commission merchant sells coal to the amount of $ 550 ; what is his commission, at 2 per cent. ?

OPERATION.

$ 550
.02
Ans. $ 11.00

Since the commission on $ 1 is .02 of a dollar, on $ 550 it will be 2 hundredths of $ 550, or $ 550 × .02, which is $ 11. Therefore, his commission on $ 550, at 2 per cent., is $ 11.

Rule.

Multiply the given sum by the number of per cent., expressed decimally.

What is a Commission Merchant, or Agent? A Broker? What is Commission? Brokerage? Explain the operation. Repeat the Rule.

Examples.

2. How much must be paid for exchanging $ 350 uncurrent bills, at 3 per cent. brokerage?

Ans. $ 10.50.

3. An agent sells goods to the amount of $ 1244; what is his commission, at 2½ per cent.?

4. An auctioneer sold property to the amount of $ 1780; what were his fees, at 1¼ per cent.?

PROFIT AND LOSS.

204. Profit and Loss is the process of estimating the gain or loss on capital employed in business.

205. To find the amount of profit or loss at any per cent.

1. If a merchant buys goods to the amount of $ 1248, and sells them at a profit of 20 per cent., how much does he gain?

OPERATION.

$ 1248
.20
———
Ans. $ 249.60

Since the profit is 20 per cent. of $1248, it must be 20 hundredths of $ 1248, or $ 1248 × .20, which is $ 249.60. Therefore, the gain on $ 1248, at 20 per cent. profit, is $ 249.60.

Rule.

Multiply the number upon which the percentage is reckoned, by the number of per cent., expressed decimally.

What is Profit and Loss? Repeat the Rule.

Examples.

2. A cistern containing 120 gallons lost by leakage 5 per cent. of its contents; how much was the loss? Ans. 6 gallons.

3. If United States stocks bought for $ 300 should advance 9½ per cent., what would be the gain?
Ans. $ 28.50.

4. Bought a horse for $ 216, and sold him at a loss of 10 per cent.; how much was the loss?
Ans. $ 21.60.

206. To find the rate per cent. of gain or loss.

1. If I buy a barrel of flour for $ 8 and sell it for $ 10, what is the gain or loss per cent.?

OPERATION.

$$\$10 - \$8 = \$2;\ \frac{\cancel{2} \times 100}{\underset{4}{\cancel{8}}} = 25, \text{ Ans.}$$

$ 10 — $ 8 = $ 2, the sum gained. Then, since 8 is 100 per cent. of itself, 1 is ⅛ of 100 per cent., and 2 is $\frac{2}{8}$ of 100 per cent., which is ¼ of 100 per cent., or 25 per cent. Therefore, if I buy a barrel of flour for $ 8 and sell it for $ 10, I gain 25 per cent.

Rule.

Multiply the number denoting the gain or loss by 100, *and divide by the number on which the gain or loss is reckoned.*

Explain the operation. Repeat the Rule.

Examples.

2. Bought a cord of wood for $6, and gained in selling it $.72; what was the gain per cent.?

3. When molasses is bought for 90 cents a gallon, and sold at 108 cents, what is the gain per cent.?

4. Bought a horse for $125, and sold it for $12.60 less than cost; what per cent. was the loss?

5. If I sell a farm which cost me $5000 for $4700, what is the loss per cent? Ans. 6 per cent.

INTEREST.

207. Interest is percentage allowed for the use of money, or for value received.

208. The **Principal** is the sum on interest.

209. The **Rate** of interest is the per cent. allowed on the principal for one year, or any given time.

210. The **Amount** is the sum of the principal and interest.

211. Legal Rate is the rate per cent. established by law. The legal rate in most of the States and in the United States Courts is 6 per cent.

In New York, Michigan, Wisconsin, Minnesota, Iowa, South Carolina, and the parts of New Jersey near New York City, it is 7 per cent.; in Louisiana it is 5 per cent., in a few States 8 per cent.; and in some States as high as 10 per cent.

The rate for one year, and at 6 per cent., is to be understood in this book when no other is named.

What is Interest? The Principal? The Rate? The Amount? Legal Rate? What rate is to be understood when no other is named?

212. Since the interest of $1, at 6 per cent. for 1 year, or 12 months, is 6 cents, for 1 month it will be $\frac{1}{12}$ of 6 cents, which is 5 mills, and for 2 months, 2 times 5 mills, or 1 cent; also,

Since the interest for 1 month, or 30 days, is 5 mills, the interest for 6 days, or $\frac{1}{5}$ of 30 days, will be 1 mill, and for 2 days, 3 days, etc., as many sixths of a mill as there are days. Hence, the

Table.

Interest of $1, at 6 per cent.,

For	12 months,	or	1 year	is	$0.06
"	2 months,	"	$\frac{1}{6}$ year	"	0.01
"	1 month,	"	$\frac{1}{12}$ year	"	0.005
"	6 days,	"	$\frac{1}{5}$ month	"	0.001
"	1 day,	"	$\frac{1}{30}$ month	"	0.000$\frac{1}{6}$.

Case I.

213. To find the interest of any sum at 6 per cent.

1. What is the interest of $212 for 2 y. 7 mo. 15 da.?

OPERATION.

Interest of $1,		Principal, $212
For 2 years	= $.12	.157$\frac{1}{2}$
" 7 months	= .035	
" 15 days	= .002$\frac{1}{2}$	1484
		1060
	$.157$\frac{1}{2}$	212
		106
		Ans. $33.390

Repeat the Table.

Since the interest of $ 1 for one year is 6 cents, for 2 years it is 12 cents, for 7 months ½ of 7 cents, or 3 cents 5 mills, and for 15 days, ⅙ of 15 mills, or 2½ mills; and taking the sum, we have the interest for 2 y. 7 m. 15 da., $ .157½. If the interest of $ 1 is $ .157½, the interest of $ 212 is 212 times $ .157½, which is $ 33.39. Therefore, the interest required is $ 33.39.

Rule.

Find the interest of $ 1 for the given time, by reckoning for YEARS *six times as many cents as there are years; for* MONTHS *one half as many cents as there are months; and for* DAYS *one sixth as many mills as there are days; and multiply the principal by the number denoting that interest.*

Examples.

What is the interest

2. Of $ 187.25 for 1 year, 4 months? Ans. $ 14.98.
3. Of $ 98.20 for 1 year, 28 days?
4. Of $ 126.46 for 9 months? Ans. $ 5.69.
5. Of $ 194 for 4 months, 12 days? Ans. $ 4.268.
6. Of $ 318 for 10 months, 16 days?
7. Of $ 320 for 33 days? Ans. $ 1.76.
8. Of $ 500 for 93 days? Ans. $ 7.75.
9. Of $ 620 for 2 years, 3 months, 3 days?
10. Of $ 812 for 3 years, 6 months, 6 days? Ans. $ 171.332.

Explain the operation. Repeat the Rule.

Case II.

214. To find the interest of any sum at any rate.

1. Required the interest of $640 for 3 years, 7 months, and 3 days, at 7 per cent.

FIRST OPERATION.		SECOND OPERATION.
$640		$640
.215½		.07
3200	Int. 1 y.,	$44.80
640		3
1280		
320	Int. 3 y.,	$134.40
	Int. ½ y., or 6 mo.,	22.40
6) $137.920	Int. ⅙ of 6 mo., or 1 mo.,	3.733+
22,986+	Int. ⅒ mo., or 3 da.,	.373+
Ans. $160.906+		Ans. $160.906+

Since the interest of $1 for the given time, at 6 per cent., is $.215½, the interest of $640 is 640 times $.215½, which is $137.92; and one sixth of this, or $22.986+, is the interest at 1 per cent., which added to the interest at 6 per cent., gives $160.906+, as the interest at 7 per cent.

Again, since the interest of $1 for 1 year, is $.07, the interest of $640 for 1 year is 640 times $.07, which is $44.80, and for 3 years is 3 times $44.80, which is $134.40; 7 months equal 6 months plus 1 month; and since 6 months equal ½ of a year, the interest for 6 months is ½ of $44.80, which is $22.40; and since 1 month is ⅙ of 6 months, the interest is ⅙ of $22.40, which is $3.783+; and since 3 days are ⅒ of a

---

Explain the first operation. The second.

month, the interest for 3 days is $\frac{1}{10}$ of $ 3.733+, which is $ .373+; and, taking the sum of these, we have as the whole interest $ 160.906+.

Therefore, the interest of $ 640 for 3 years, 7 months, and 3 days, at 7 per cent., is $ 160.906+.

Rule.

Find the interest for YEARS *by multiplying the principal by the rate expressed decimally, and that product by their number; for* MONTHS, *take such a part of the interest for one year as their number is part of a year; and for* DAYS, *take such a part of the interest of one month as their number is part of a month.* Or,

Find the interest at 6 *per cent., and increase or diminish as the per cent. is greater or less than* 6 *per cent.*

For 7 per cent., add to 6 per cent. $\frac{1}{6}$; for 8 per cent., $\frac{1}{3}$; for 9 per cent., $\frac{1}{2}$; for 5 per cent. subtract $\frac{1}{6}$; for 4 per cent., $\frac{1}{3}$, etc.

Examples.

2. What is the interest of $ 780 for 5 years, at 7 per cent.? Ans. $ 273.

3. What is the interest of $30.50 for 3 years at 5 per cent.? Ans $ 4.575.

4. What is the interest of $ 130.60 for 6 months, at 7 per cent.? Ans. $ 4.571.

5. What is the interest of $ 516 for 9 months, at 7 per cent.?

What is the Rule? For 7 per cent., what is added to 6 per cent.? For 8 per cent.? For 9 per cent.? For 5 per cent., what is subtracted? For 4 per cent.?

6. What is the interest of $234.92 for 33 days, at 7 per cent.? Ans. $1.507+.

7. What is the interest of $420 for 63 days, at 9 per cent.? Ans. $6.615+.

8. What is the interest of $226 for 2 years and 3 months, at 5 per cent.? Ans. $25.425.

9. What is the interest of $144.20 for 3 years and 8 months, at 10 per cent.? Ans. $52.873.

10. What is the interest of $210, for 4 years and 5 months, at 7 per cent.?

11. What is the interest of $67.42, for 1 year, 7 months, and 15 days, at 6 per cent.? Ans. $6.573+.

12. What is the interest of $333, for 5 years, 2 months, and 5 days, at 8 per cent.? Ans. $138.01.

215. When the amount is required, add the principal and interest. (Art. 210.)

13. What is the amount of $263.48, for 2 months and 21 days, at 6 per cent.? Ans. $267.036+.

14. What is the amount of $54.98 for 6 months and 20 days, at 7 per cent.?

15. What is the amount of $45.15 for 1 year, 4 months, and 9 days, at 5 per cent.? Ans. $48.216+.

16. What is the amount of $60.60, for 2 years, 5 months, and 16 days, at 8 per cent.? Ans. $72.531+.

17. What is the amount of $100 from May 16, 1865, to August 31, 1867, at 10 per cent.? Ans. $122.916+.

REVIEW QUESTIONS.—What is Percentage? (194.) What is Commission? (200.) Brokerage? (201.) Profit and Loss? (204.) *Interest? (207.)*

Miscellaneous Examples.

1. A merchant receives from the sale of coffee $519.60, of tea $310.20, and of sugar $640.45; how much does he receive from the whole? Ans. $1470.25.

2. If you should buy a farm for $5600, and sell a part of it for $3780, and the remainder for $2340, how much would you make by the operation?

3. The difference of two numbers is 397, and the smaller number is 532; what is the larger number?

4. A and B together had $655; but A having lost $65, they then had equal sums; how much money had B? Ans. $295.

5. A merchant bought 55 bales of cloth, each bale containing 36 pieces, and each piece 30 yards; how many yards did he buy in all? Ans. 59400.

6. Two travelers set out from towns 363 miles distant, and travel towards each other till they meet, when it appears one has traveled 93 miles farther than the other; how far had each traveled?
Ans. One, 135; the other, 228 miles.

7. If the product of two numbers is 1960, and one of the numbers is 35, what is the other number?

8. If I have $3642, and purchase 404 barrels of flour at $9 a barrel, how much shall I have left after paying for the flour? Ans. $6.

9. How many bushels of wheat at $1.25 a bushel will pay for 10 yards of broadcloth at $4.50 per yard? Ans. 36 bushels.

REVIEW QUESTIONS.—What is Quantity? (10.) Numbers? (11.) Arithmetic? (15.) Figures? (16.) Notation? (32.) Numeration? (36.) Upon what principle is Arabic Notation based? (33.)

10. Divide the product of 35 × 20 by 640 — 515.

11. If 9 acres of grass will pasture 15 cows, how many cows will 6 acres pasture?

12. When 141 tons of coal can be bought for $ 775.50, how much is the cost of 100 tons?

13. Required the prime factors of 1365.
Ans. 3, 5, 7, and 13.

14. Reduce $312\frac{4}{5}$ to an equivalent improper fraction.

15. Reduce 1564 to an equivalent fractional form.

16. Change $\frac{3}{5}$ and $\frac{4}{7}$ to equivalent fractions having a common denominator. Ans. $\frac{21}{35}$ and $\frac{20}{35}$.

17. A merchant sold, at one time, $\frac{2}{5}$ of a ship, at another time, $\frac{1}{2}$ of the remainder; what part of the whole ship did he sell at both times? Ans. $\frac{7}{10}$.

18. $\frac{2}{5} + \frac{3}{10}$ subtracted from 1 leaves how many?

19. What is the cost of 6 acres and 2 roods of land, at $ 12.50 a rood? Ans. $ 325.

20. What is the difference of time between Jan. 30, 1865, and May 16, of the same year?

21. How many $1\frac{1}{2}$ pint bottles can be filled from a cask of wine containing 54 gal. 1 qt. 1 pt.?
Ans. 290 bottles.

22. Gave $ $4\frac{5}{8}$ dollars for a vest, $ $10\frac{3}{4}$ for a coat, and $ $5\frac{3}{16}$ for boots. How much did the whole cost?

23. At $\frac{3}{8}$ of a dollar a yard, how many yards of cloth can be had for $ 12? Ans. 32 yards.

24. What decimal is equivalent to $\frac{15}{16}$?

Review Questions.—Name the orders in the Numeration Table? (34.) What is the Rule for Addition? (41.) For Subtraction. (48.) For Multiplication? (57.) For Short Division? (68.) For Long *Division? (70.)*

25. What is the difference between thirty thousandths and thirty thousand? Ans. 29999.970.

26. Purchased two loads of hay, the first weighing 16 cwt. 3 qr. 14 lb., and the second 17 cwt. 2 qr. 11 lb. What was the weight of the two loads?

Ans. 1 T. 14 cwt. 2 qr.

27. If 105 is $\frac{3}{7}$ of some number, what is that number? Ans. 245.

28. When $5500 is the value of $\frac{1}{2}$ of $\frac{5}{8}$ of a cotton factory, what is the whole of it worth?

29. How long will 2295 pounds of beef last a garrison of 45 men, if each man should get 1 pound 3 times a week? Ans. 17 weeks.

30. A farmer, being asked how many sheep he had, answered, that he had them in 4 fields; in the first he had $\frac{1}{4}$ of his flock, in the second $\frac{1}{5}$, in the third $\frac{1}{6}$, and in the fourth 46; how many had he in all?

Ans. 120.

31. What common fraction is equivalent to .9375?

32. What per cent. of 60 is 15?

33. What is the quotient, when 500 is the dividend and .05 the divisor? Ans. 10000.

34. Required the quotient of .05 divided by 500.

35. What is the product of .0001 multiplied by 500? Ans. .05.

36. If the number of acres in a certain town is 12,935, and the total valuation of real estate is $714,200, what is the average value of real estate to the acre? Ans. $55.21+.

REVIEW QUESTIONS.—What is United States Money? (71.) How may United States Money be written, added, etc.? (75.) What is Reduction? (112.) What is the Rule for Reduction Descending? (114.)

37. What is an auctioneer's commission for selling goods to the amount of $ 16000, at 2¼ per cent. ?

38. A farmer divided his real estate, consisting of 113 A. 3 R. 25 P., equally among his 9 children; how many acres did each receive ?

Ans. 12 A. 2 R. 25 P.

39. How much wood is contained in a load 8 feet long, 3½ feet wide, and 5½ feet high ?

Ans. 1 C. 26 cu. ft.

40. If ⅞ of a ship cost $ 35000, what is ½ of it worth ? Ans. $ 20000.

41. How many yards of cloth 1¼ yards wide will line 16 yards ⅜ of a yard wide ? Ans. 4⅘ yards.

42. Bought a horse for $ 125, but not proving as good as expected, I sold him for $ 110. What was the loss per cent. ? Ans. 12 per cent.

43. Bought a house for $ 3600, and sold it at a loss of 7 per cent. ; how much did I obtain for it ?

Ans. $ 3348.

44. Required the interest on $ 27.60 for 6 years, 6 months, and 6 days, at 6 per cent.

Ans. $ 10.791+.

45. If the interest of $ 350 for 1 year is $ 21, what is the interest of $ 250 for the same time ?

46. What number multiplied by 12½ gives 1000 ?

47. What number divided by 7¾ gives 110 ?

48. If I buy corn for 50 cents a bushel, and sell it at an advance of 20 per cent., how many cents do I gain per bushel ?

Review Questions.—Reduction Ascending? (117.) What is a Simple Number? (119.) A Compound Number? (120.) A Factor? (130.) Factoring? (132.) Cancellation? (135.) Analysis? (137.)

49. The sum of two numbers is 93, and one of them is $11\frac{1}{5}$; what is the other?

50. If I buy corn for 60 cents a bushel, and sell it at a loss of 20 per cent., what do I get a bushel for it?

51. Of a certain pole, 5 feet stands in earth, 10 feet in water, and $\frac{2}{5}$ above water; what is the length of the pole? Ans. 25 feet.

52. What fraction is that to which if you add $\frac{5}{7}$ the sum will be $\frac{27}{28}$? Ans. $\frac{1}{4}$.

53. How many rectangular blocks $1\frac{1}{2}$ feet wide and 2 feet long, will pave a cellar-floor 15 feet wide and 16 feet long? Ans. 80 blocks.

54. When the dividend is 17.28, and the quotient 14.4, what is the divisor? Ans. 1.2.

55. When the dividend is .001, and the quotient .000001, what is the divisor? Ans. 1000.

56. Bought a horse, and paid $ 72 down, which was $\frac{6}{11}$ of the price; what was the price? Ans. $ 132.

57. What is the interest on $ 600 for 6 months, at $7\frac{3}{10}$ per cent?

58. Bought a cask of molasses, containing 65 gallons, for $ 26; for how much per gallon must I sell it to gain $12\frac{1}{2}$ per cent.? Ans. $ .45.

59. What is the value of a pile of wood 36 feet long, 4 feet wide, and 6 feet high, at $ 6 per cord? Ans. $ 40.50.

60. How many steps, of 2 feet 6 inches each, must a man take, in walking a mile? Ans. 2112 steps.

Review Questions. — What is a Fraction? (139.) A Common Fraction? (142.) A Decimal Fraction? (172.) From what is the term per cent. derived? (194.)

61. If the cost of $\frac{7}{8}$ of a farm be $3500, what is $\frac{3}{4}$ of it worth? Ans. $3000.

62. What is the value of a small rectangular farm, 72 rods long and 40 rods wide, at $50 per acre?

63. A frog, at the bottom of a well 15 feet deep, commenced going towards the top. In his journey he got up 3 feet every day, and fell back 2 feet every night. How many days did it take him to get out of the well?

64. What is the interest on a note of $500 from Jan. 28, to May 21, 1867, at 7 per cent.?

Ans. $10.986+.

65. A man purchased a hat worth $5, and handed the merchant a $50 bill in payment; but the merchant, not being able to make change, passed it to a broker, and on getting it changed, he gave the purchaser of the hat his change. After the customer had left, the broker, finding the bill to be counterfeit, returned it to the merchant, and received good money for it. How much did the merchant lose by the transaction?

66. A farmer being asked how many sheep he had, said, "if you add to $\frac{1}{2}$ of the number, $\frac{1}{2}$ of $\frac{3}{4}$ of it, and 5 sheep, you will have the number;" how many had he? Ans. 40.

67. The population of the United States in 1860 was 31,000,000, and if the increase be at the rate of 30 per cent. for the next 10 years, what will the population amount to in 1870? Ans. 40,300,000.

Review Questions.—What is Commission? (200.) Profit and Loss? (204.) What is Interest? (207.) What is the Rule for finding *the interest* on any sum, for any rate? (214.)

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